Tournaments with prize-setting agents[*]

Kristoffer W. Eriksen[†], Ola Kvaløy[‡] and Trond E. Olsen[§]

October 28, 2009

Abstract

In some tournaments it is the contestants themselves who determine reward allocation. Union members bargain over wage distribution, and some firms allow self-managed teams to freely determine internal resource allocation, incentive structure and division of labour. We analyze, and test experimentally, a rank-order tournament where heterogenous agents determine the spread between winner prize and loser prize. We investigate the relationship between prize spread, uncertainty (i.e. noise between effort and performance), heterogeneity and effort. The paper challenges well-known results from tournament theory. We find that a large prize spread is associated with low degree of uncertainty and high degree of heterogeneity, and that heterogeneity triggers effort. By and large, our real-effort experiment supports the theoretical predictions.

---

[*] We thank the editor and three referees for constructive comments. We also thank Bård Harstad, Hans Hvide, Frode Meland, Espen Moen, Klaus Mohn, Mari Rege, Christian Riis, Eirik Romstad, Gaute Torsvik, Ragnar Tveterås, Sigbjørn Tveterås and seminar participants at the Norwegian School of Management, ESOP workshop at the University of Oslo, the Conference of the Norwegian Economic Association, the Nordic Workshop of Behavioral and Experimental Economics, and the Conference of the European Economic Association for helpful comments and discussions. We thank Marius Sikveland for excellent research assistance, and the Norwegian Research Council for funding.

[†] University of Stavanger, 4036 Stavanger, Norway. E-mail: kristoffer.w.eriksen@uis.no

[‡] University of Stavanger, 4036 Stavanger, Norway. E-mail: ola.kvaloy@uis.no

[§] Norwegian School of Economics and Business Administration, Helleveien 30, 5045 Bergen, Norway. trond.olsen@nhh.no

1 Introduction

In many areas of economic, political and social life, "the rules of the game" are determined by its players: Politicians determine rules of elections, sports federations determine rules for leagues and tournaments, and the allocation of resources within firms and organizations is sometimes decided by its members/employees.

Tournament theory provides us with a tool for analyzing these phenomena. The theory was first introduced by Lazear and Rosen (1981) as an effort to understand situations where wage differences are based on relative differences between the individuals rather than on marginal productivity. The theory has had enormous impact. In many settings, tournaments are found to be at least as good as any other incentive mechanism in terms of inducing effort, and comparative static results on the optimal tournament solution have provided insights into internal wage policies of firms.[1]

So far tournament theory has not been used to analyze games where the players set the rules. In particular, it is always assumed that the spread between winner prize and loser prize (we use the term prize spread throughout this paper) is determined by a non-participating principal. But there are important examples where prize spreads are (explicitly or implicitly) set by the contestants themselves.

One example is related to self-management and democracy in organizations. An increasing number of firms allow self-managed teams to determine internal resource allocation, reward structure and division of labour

---

[1] By tying compensation to the agent's relative performance, the principal can filter out common noise so that compensation to the largest possible extent is based on real effort, not random shocks that are outside the agent's control (see Holmström,1982; and Mookherjee, 1984). With RPE's special form, rank-order tournaments, the agents are also completely insulated from the risk of common negative shocks (see Lazear and Rosen, 1981; Green and Stokey, 1983).

(Osterman, 1995; and Jehn et al.,1999).[2] And in democratic organizations, members or employees often vote over strategic decisions that involve rules for internal resource allocation. Academia is a good example of this. The distribution of resources within research groups, or the incentive structure within research departments are often decided by the members of the group or department.

Another important example is decentralized wage bargaining. The last two decades have seen a trend in OECD countries towards more decentralized (local) bargaining between firms and labor unions (see Cahuc and Zylberberg, 2004). Wage dispersion will typically be higher under decentralized bargaining since individual-specific characteristics are more likely to enter the wage contracts (Dahl et al. 2009). When unions have bargaining power, their preferences and wage policy become important. The union may emphasize performance rather than seniority, or it may have preferences for equal raises rather than higher wage dispersion. Under decentralized wage bargaining, a coordinated process within the union determines its wage policy. This is in essence agents bargaining over a fixed prize pool. Even if it is a kind of ex post bargaining, it also sets the norms and rules for future allocations.[3]

In this paper we analyze and test experimentally a rank-order tournament where heterogeneous agents determine prize spreads. The theoretical model assumes risk neutrality, and the low-ability agent (he) will then clearly prefer zero prize spread in such a tournament. For the high-ability agent

---

[2] See Yeatts and Hyten (1997) for an analysis of the emergence of self-managed teams in modern corporations.

[3] A quite common procedure in some sectors, especially in Scandinavia, is that the union members report their wage demand to the local union leader, and after a coordinated process within the union, the union sets up a priority list. The union also hands over the individual wage demands to the employer, who makes its own priority list. The employer and the union then bargain over how to distribute the wage raises (see Seip, 2002 and Stokke and Seip, 2003, for details).

(she), however, determining optimal prize spread is not straightforward. A high prize spread is good since she expects to win. But it is bad since it triggers effort, and effort is costly. Since the optimal prize spread for the low-ability-agent is always zero, we can focus on the high-ability agent's optimal choice. We characterize her optimal prize spread and investigate the relationship between prize spread, uncertainty (i.e. noise between effort and performance), heterogeneity and effort.

Theoretical results: First, we find that the high-ability agent's optimization problem entails corner solutions. Either she wants zero prize spread or she wants maximal prize spread. A small parameter change may thus dramatically change prize spread and effort. This is interesting since it can explain why seemingly similar firms may differ substantially in wage structure and performance (see Gibbons et al., 2007, for a discussion on persistent performance differences among seemingly similar enterprises).

Second, we find that more heterogeneity (i.e. larger ability-difference) leads to higher equilibrium effort. This is an interesting result since it challenges theory stating that heterogeneity reduces effort. In Lazear and Rosen (1981), effort suffers from more heterogeneity, or at best is unaffected by ability-difference if the principal can observe the agents' type so that she can perfectly compensate heterogeneity with higher prize spread. We show that higher ability-difference increases prize spread *more* than just to compensate for heterogeneity, leading to higher equilibrium effort.

Third, we find that large prize spread is associated with low degree of uncertainty. This contrasts with the standard tournament result where the optimal prize spread increases with uncertainty. Our result is not trivial, since there are two countervailing effects: As uncertainty increases, the probability of winning decreases cet. par. so the high-ability agent might want to decrease prize spread in order to reduce effort costs. However, the high-ability agent can 'remove' the reduced winner probability by increasing the prize spread, since higher prize spread increases the effort-difference between the agents. We show that the former effect dominates under standard assumptions.

From an incentive perspective, the result offers an alternative explanation to a negative relationship between uncertainty and incentives. The standard explanation is risk aversion; the optimal intensity of incentives is negatively related to uncertainty when agents are risk averse.[4] Our model shows that tournaments with prize-setting agents can create such a relationship even if agents are risk neutral.

The result also points to the issue of "desert", or whether or not performance pay is "fair". According to Konow (2003), a common view is that differences owing to luck are unfair, and that only differences attributable to effort are fair. Our high-ability agent seemingly has fairness concerns since her preferences are aligned with the firm's preferences for high prize spread if effort is important. But if luck is important, then her preferences are aligned with the low-ability employee. However, this is not because of fairness concerns; she simply makes a trade-off between effort costs and expected monetary payoff. One should thus be cautious with drawing the conclusion that employees have fairness concerns if they argue that uncertainty makes performance pay unfair.

Experimental results: We conjecture that comparative static results on the high-ability agent's optimal prize spread also apply for the bargaining solution between the agents. We conducted a real effort experiment to test this conjecture for some of our theoretical results. We elicited subjects'

---

[4] Prendergast (2002) shows, though, that the relationship between uncertainty and incentives may be positive.

risk preferences and their ability to do head calculation, and we then got them to bargain over winner and loser prizes prior to two-player tournaments in head calculation. This enabled us to test the relationship between prize spread, ability-difference and effort. We also imposed two uncertainty levels, high and low, enabling us to study the relationship between prize spread and noise. By and large, the experiment supports the theoretical predictions. First, controlling for risk preferences, we find that prize spread significantly decreases with uncertainty, which supports the theoretical prediction. Second and third, we find that prize spread significantly increases with increasing ability-difference between the agents, and that exerted effort increases with increasing prize spread. This supports the theoretical prediction that more heterogeneity increases prize spread, which thereby increases effort. Controlling for prize spread, however, we find a significant negative relationship between ability-difference and effort, supporting previous experimental results.

Related literature: As noted above, neither the tournament literature nor the union literature have analyzed rank-order tournaments where heterogenous agents set the prize spread.[5] Brunello (1994) analyzes a case where homogenous agents decide prize spread in a principal-agents game with a flexible wage pool; and Sutter (2006) analyzes an endogenous prize selection tournament where the best member of a team is given a right *ex post* to propose prize spread within the team. Neither of these papers analyzes a situation where heterogenous contestants determine prize spread prior to the tournament. Riis (2007) analyzes a tournament where heterogenous contestants can choose from a menu of prizes, but the prize menu is defined

---

[5] Unions composed of identical members has been the basis for representations of union preferences (see Cahuc and Zylberberg, 2004), although Ross (1948) already 60 years ago argued that the heterogenity of union members affects its aims. See also Boot (1984) on the importance of heterogenous preferences in unions.

by the principal ex ante. And while Riis focuses on how the principal can structure the prize menu so as to implement first-best effort, we focus on the agents' optimal prize spread and the comparative statics that can be derived from the agents' solution.

Several papers have experimentally tested hypotheses deduced from tournament theory, starting with Bull et al. (1987).[6] Typically, these papers test the relationship between prize spread, effort and heterogeneity. But there are only a few real effort experiments testing the theory (van Dijk, Sonneman and van Winden, 2001, and Dohmen and Falk, 2006), and no one has examined a case where the agents set prize spread - although Sutter (2006) runs an experiment (not real effort) where he tests his model of endogenous prize selection.

The rest of the paper is organized as follows: Section 2 presents the model, Section 3 outlines the experimental design, Section 4 formulates the hypothesis to be tested, Section 5 presents results and analysis from the experiment, while Section 6 concludes. Proofs and tables are in the appendix.

2 The model

Consider a tournament between two risk-neutral agents. The winner of the tournament receives w_1 while the loser receives $w_2 \leq w_1$. Output y_i from agent i is given by

$$y_i = e_i + z_i$$

where e_i is effort and z_i is a random luck component. Expected payoff for agent i is

$$Pw_1 + (1 - P)w_2 - C_i(e_i) \tag{1}$$

---

[6] See Harbring and Irlenbusch (2004) for an overview of these experiments.

where $C_i(e_i)$ is the cost of effort ($C_i' > 0 \, and \, C_i'' > 0$) and P is the probability of winning: $P = prob(y_i > y_j) = prob(e_i - e_j > z_j - z_i) = G(e_i - e_j)$. Here $G(.)$ is the cumulative distribution function of the random variable $z_j - z_i$, with density $g(.)$; and we assume $G(e_i - e_j) = 1 - G(e_j - e_i)$. For given prizes, the players simultaneously choose effort to maximize their respective expected payoffs (1). This gives the IC constraint (for interior solution)

$$(w_1 - w_2)\frac{\partial P}{\partial e_i} = C_i'(e_i) \tag{2}$$

From Nash-assumptions it follows that $\frac{\partial P}{\partial e_i} = \frac{\partial G(e_i - e_j)}{\partial e_i} = g(e_i - e_j)$. The IC constraint (2) is thus

$$(w_1 - w_2)g(e_i - e_j) = C_i'(e_i) \tag{3}$$

We invoke the standard assumption that the total prize pool is fixed, i.e. that $w_1 + w_2 = R$, where R is exogenous and unaffected by effort levels, but we discuss later how a change in R affects prize spreads. A fixed R may appear like a strict assumption, but in many tournaments the prize pool is indeed fixed. In pure promotion tournaments for example, the sum of prizes is unaffected by effort-levels. And in larger bureaucratic organizations, total resource provision to organizational divisions may often be exogenously given, or at least perceived as exogenous by the employees. Decentralized wage bargaining is also a good example. In many countries, the *size* of the wage pool that is to be allocated locally in each firm is determined by central bargaining between labor unions and employer federations. The size of the local wage pool is then unaffected by total effort levels, and the only thing bargained over locally is distribution of the fixed wage pool.[7]

---

[7] Note also that if the absolute value of output is unverifiable to a third party, then a

Consider the following timing. First, for a fixed pool R one agent (say i) sets the two prizes w_1, w_2 subject to $w_1 + w_2 = R$. In a second stage the two agents play a tournament with these prizes. For a fixed pool, the prize spread is $w_1 - w_2 = R - 2w_2$, and effort levels e_i, e_j are then determined by (3) as functions of w_2 in the tournament. Agent $i's$ optimal prize spread is then the solution to

$$\max_{w_2} [w_2 + P(e_i(w_2), e_j(w_2))(R - 2w_2) - C_i(e_i(w_2))]$$

With identical (homogenous) agents, the equilibrium success probability in the tournament is $P = \frac{1}{2}$, hence the expected prize for each agent is $\frac{1}{2}R$ and does not vary with the prize spread. The agent will then simply minimize effort costs, which is achieved by setting zero prize spread, $R - 2w_2 = 0$, such that the equilibrium effort level is zero. This is the collusion logic, first thoroughly analyzed by Mookherjee (1984). When prizes are fixed, agents have incentives to collude on low effort equilibria. It follows here that if they are to decide prizes, they set them so that effort is minimized.

Heterogeneous agents: A tournament model with prize-setting agents first becomes interesting when there is heterogeneity in ability levels. Of course, the agents still have incentives to collude on zero prize spread by using side payments. It is, however, a quite standard assumption in the tournament literature to assume that collusive contracts are not enforceable. We thus invoke the assumption that side payments are not feasible.

We model differences in ability levels by assuming that the marginal cost from effort is lower for the high-ability agent i, i.e. $C_i'(e_i) < C_j'(e_j)$ for all

---

fixed prize pool may turn out optimal: With a flexible prize pool, total prize payments increase in effort, making the principal's incentive to renege on payments increase in effort. A fixed prize pool removes this problem, and makes it easier for the principal to commit to prize promises (see e.g. Carmichael, 1983).

$e_i = e_j$. These costs are common knowledge. A symmetric density function $g(e_j - e_i) = g(e_i - e_j)$ implies that $C_i'(e_i) = C_j'(e_j)$ in equilibrium, which for interior solutions implies higher effort and higher winning probability for the high-ability agent ($e_j < e_i$ and thus $\frac{1}{2} < G(e_i - e_j) < 1$).

With no restrictions on prizes, a prize-setting principal can achieve first-best when ability levels are common knowledge. For prize-setting agents, however, first-best implementation is not the objective if the wage pool is fixed in advance.

From the restriction $w_1 \geq w_2$, it is straight-forward to see that the optimal prize spread for the low-ability agent j is zero. He has nothing to gain from increasing prize spread, since this implies costly effort and a reduced chance of winning the tournament. Hence,

Proposition 1 *If $w_1 \geq w_2$, and $C_i'(e_i) < C_j'(e_j)$ for all $e_i = e_j$, then the low ability agent $j's$ optimal prize spread is zero, yielding zero effort in equilibrium.*

Since optimal prize spread for agent j is always zero, we can now concentrate on deducing the optimal prize spread for the high-ability agent i. We do not analyze a general bargaining game between the agents, but we conjecture that if the high-ability agent has some bargaining-power, then comparative static results on her optimal prize spread hold qualitatively for the bargaining solution between the agents.[8]

As noted in the introduction, we assume that the parties cannot use side payments in order to collude on low effort / zero prize-spread. Taking this

---

[8] Note that if agent i's bargaining power is sufficiently strong, her optimal choice will in fact be the outcome of a bargaining game between the agents. This is the case if, for instance, i proposes a spread which j must accept or refuse, and the principal decides and sets a maximal spread (to induce highest effort for the given R) if j refuses $i's$ proposal. It is not unrealistic that the best agent has the strongest bargaining power, since she also normally has the best outside options.

into account, if the high-ability agent i sets the prize spread, she solves

$$\max_{w_2} W(w_2) = [w_2 + G(e_i(w_2) - e_j(w_2))(R - 2w_2) - C_i(e_i(w_2))] \tag{4}$$

Agent i will choose $(R - 2w_2) > 0$ if there exist equilibrium effort levels e_i, e_j where[9] $G(e_i - e_j)(R - 2w_2) > C_i(e_i)$. For the rest of the paper, we assume that the ability-difference is sufficiently large such that this condition holds in equilibrium. This is not a strict assumption.[10]

From (4) we obtain, using (3):

$$W'(w_2) = 1 - 2G(e_i - e_j) - (R - 2w_2)g(e_i - e_j)\frac{\partial e_j}{\partial w_2} \tag{5}$$

Equation (5) shows that the marginal value for agent i of increased w_2 (reduced prize spread) has two components. First, for given efforts her expected payment is reduced. This marginal payment effect is $1 - 2G(e_i - e_j) < 0$, where the inequality follows from the fact that she will exert higher equilibrium effort $(e_i > e_j)$ due to her ability advantage, and hence will win with a probability exceeding 1/2. Second, there is an indirect effect induced by reduced effort on the part of the other agent $(\frac{\partial e_j}{\partial w_2} < 0)$, and this effect will increase agent i's probability of winning the tournament.[11] Thus, a reduced prize spread (increased w_2) yields one negative and one positive effect for

---

[9] Note that the tournament equilibrium underlying this analysis will exist only if the IC conditions for the agents' efforts reflect truly optimal choices. In particular, the second-order conditions must hold, hence we must have $(R - 2w_2)g'(e) - C_i''(e_i) \leq 0$ and $(R - 2w_2)g'(-e) - C_j''(e_j) \leq 0$, where $e = e_i - e_j$. Since $g'(e) < 0$ for $e > 0$, the first will hold for convex costs, but the second may not, since $g'(-e) = -g'(e) > 0$. It follows that the level of uncertainty has to be sufficiently large for a tournament equilibrium to exist. This also applies for standard tournament models where the principal sets prize spread (see Lazear and Rosen, 1981).

[10] With Inada conditions, i.e. $C_l(e_l)$, $l = i, j$ continuously differentiable, strictly increasing and $C_l(0) = C_l'(0) = 0$, the stated condition holds in equilibrium for arbitrarily small ability differences.

[11] The indirect effect induced by agent i's own effort response is zero due to the IC constraint.

agent i. We will show below that under reasonable assumptions either the first or the second of these effects will dominate, so that the agent will then choose either maximal spread ($w_2 = 0$) or minimal spread ($w_2 = R/2$).

Consider first the case of minimal wage spread, i.e. $w_2 = R/2$. In this case both agents will exert minimal effort ($e_i = e_j = 0$), they will win with equal probability $P = G(0) = \frac{1}{2}$, and we then see from (5) that $W'(w_2) = 0$ for $w_2 = R/2$.[12] This shows that a minimal prize spread and hence minimal effort is a candidate for an optimum.

To characterize the optimum we introduce further assumptions. In the following we assume $C_i(e_i) = k_i e_i^n$ and $C_j(e_j) = k_j e_j^n$ where $n > 1$ and $k_j > k_i$. The IC constraints imply equality of marginal costs;

$$nk_i e_i^{n-1} = nk_j e_j^{n-1} \tag{6}$$

and the difference in efforts in the tournament is then given by:

$$e \equiv e_i - e_j = \left((\frac{k_j}{k_i})^{\frac{1}{n-1}} - 1 \right) e_j \equiv \frac{1}{K} e_j \tag{7}$$

where K is defined by the last identity.

We will here show that a minimal prize spread ($w_2 = R/2$) is optimal for a range of parameters, and that the other corner solution ($w_2 = 0$) is optimal for other parameters. Moreover, for a class of distributions including the normal and uniform ones, we will show that the optimal solution is always a corner solution.

To this end assume that the noise is of the form

$$z_l = \sigma \varepsilon_l + a, \quad \sigma > 0 \tag{8}$$

---

[12] More precisely, this holds provided $\frac{\partial e_j}{\partial w_2}$ is bounded, which is the case if the elasticity of agent j's marginal cost function is bounded away from zero.

where ε_l has some fixed distribution and σ, a are constants. (This holds e.g. for all normal and uniform distributions.) The parameter σ is then a measure of the degree of uncertainty associated with an agent's performance.

We can now show (see the appendix) that if the uncertainty measured by σ is 'large' and/or the wage pool R is 'small', then it is overall optimal to induce minimal effort and spread when the degree of heterogeneity between the agents (measured by $\frac{k_j}{k_i}$) is small. We can also show that for small R and/or large σ it is optimal to induce maximal effort spread when the degree of heterogeneity is 'large'. More precisely, letting $e_{\max}$ be the largest feasible effort spread for the given R, i.e. the spread corresponding to $w_2 = 0$, we have the following:

Proposition 2 *(i) For low heterogeneity (*$\frac{k_j}{k_i} < \left(\frac{n}{n-1}\right)^{n-1}$*) we have: there is* $r_1 > 0$ *such that for* $R/\sigma^n < r_1$*, i.e. for* R *sufficiently small and /or* σ *sufficiently large, the optimal solution entails minimal effort and minimal prize spread;* $e = e_i = e_j = 0$ *and* $w_2 = R/2$*. (ii) For large heterogeneity (*$\frac{k_j}{k_i} > \left(\frac{n}{n-1}\right)^{n-1}$*) we have: there is* $r_2 > 0$ *such that for* $R/\sigma^n < r_2$ *the optimal solution entails maximal effort and prize spread;* $e = e_{\max}$ *and* $w_2 = 0$*.*

By invoking more assumptions we can be more precise:

Proposition 3 *For a class of distributions including the normal and uniform ones the following holds. (i) For low heterogeneity (*$\frac{k_j}{k_i} < \left(\frac{n}{n-1}\right)^{n-1}$*) we have: the optimal solution entails either (a) minimal effort and prize spread (*$e = 0$ *and* $w_2 = R/2$*) or (b) maximal effort and prize spread (*$e = e_{\max}$ *and* $w_2 = 0$*). There is* $r_1 > 0$ *such that the former is optimal for* $R/\sigma^n < r_1$*, and the latter is optimal for* $R/\sigma^n > r_1$ *(provided the tournament equilibrium exists for this case). (ii) For large heterogeneity (*$\frac{k_j}{k_i} > \left(\frac{n}{n-1}\right)^{n-1}$*) we have:*

for all parameters R, σ for which the tournament equilibrium exists, the optimal solution entails maximal effort and prize spread; $e = e_{\max}$ and $w_2 = 0$.
(iii) When the solution entails maximal spread we have $e_{\max} = \sigma d_m(R/\sigma^n)$, where $d_m'() > 0$. Effort is then increasing in R and non-monotone in σ (increasing for σ small and decreasing for σ large).

The proposition shows that the high-ability agent's optimal prize spread is high for low uncertainty (low σ) and low for high uncertainty. Hence, the standard result that prize spread increases in noise when agents are risk neutral is *not* robust to a setting where heterogeneous agents determine prize spread. We also find that prize spread and effort are low for low heterogeneity and high for high heterogeneity. This is not trivial. Higher ability-difference increases the chance of winning, cet. par. This calls for higher prize spread. But higher ability-difference makes it possible for the high-ability agent to reduce effort, and thereby reduce effort costs without affecting the probability of winning. We show that the former effect dominates under relatively general assumptions. Our results thus contrast with the well known result from tournament theory saying that performance suffers from heterogeneity.

From Proposition 3 we also see that $e_{\max}$ (and hence both efforts) is first increasing in σ, and then decreasing. Hence, the optimal uncertainty level is strictly positive. This is in contrast to standard tournaments where effort suffers from more uncertainty, or at best is unaffected by the uncertainty level if the principal can perfectly compensate noise with higher prize spread. The result complements Krakel and Sliwka (2004), who find that more noise may increase effort in a setting where agents can choose risk levels. Finally, observe that effort is increasing in R, so if the principal can control the prize pool but not the prize spread, one should expect a higher pool, R, the less heterogenous the agents are.

We have focused on the agents' optimal choices, and have not considered a general bargaining game between the agents. But if we stick to the restriction that the winner gets the highest prize ($w_1 \geq w_2$)[13] then in any bargaining game over the prize spread between agent i and agent j (prior to effort decisions), comparative statics on the optimal choice for agent i weakly hold for the bargaining solution, since the optimal spread for agent j is zero. Hence, we can state the following conjecture:

Conjecture 1 *In a bargaining game over the prize spread between agent i and agent j, we have (i) equilibrium prize spread weakly decreases in noise* (σ)*, (ii) equilibrium prize spread weakly increases in relative ability difference* $(k_j - k_i)/k_i$ *(iii) equilibrium effort weakly increases in relative ability difference* $(k_j - k_i)/k_i$*.*

In the following we will report on an experiment testing this conjecture.

3 Experimental design

The experimental design reflects our aim to investigate conjecture 1. We conducted a real effort experiment in order to make the ability-difference between the subjects natural rather than imposed. We also believe that real effort makes the meaning of noise, or luck, clearer to the subjects.

The work task for the subjects participating in the experiment consisted of doing head calculations; multiplying one- and two-digit numbers (e.g. 7 x 83).[14] The task nicely mimics real world work tasks and also ensures heterogeneity in productivity. Doing head calculations is shown to be rather

---

[13] $w_1 \geq w_2$ is a weak restriction. We did not make this restriction in our experiment, but no one ever proposed $w_2 > w_1$.

[14] The actual assignments were borrowed from Thomas Dohmen and Armin Falk who used them in Dohmen and Falk (2006).

insensitive to learning and is therefore well-suited for experimentation. A problem with real effort tasks in experiments is the potential for excessive intrinsic motivation, blurring the effect of monetary incentives. We therefore wanted to make the work task boring enough to be affected by monetary incentives. As we shall see, monetary incentives indeed affected performance, and the lack of intrinsic motivation was to some extent confirmed by the subjects' "moaning and groaning" when they learned that the experiment consisted of doing head calculations. [15]

Altogether 108 undergraduate students from the University of Stavanger were recruited by E-mail to participate in the experiment. They were told that they had the opportunity to participate in an economic experiment and if they did well they could earn a nice sum of money. The experiment was programmed in z-Tree (Fischbacher, 1999). The instructions were given both verbally and on the computer screen. The subjects were told that no form of communication was allowed throughout the experiment and that all results were to be held anonymous. We had 18 subjects in each out of six sessions. Each session lasted approximately 50 minutes. Total average earnings in the experiment were NOK 302 (38 Euro).

The subjects went through five steps. Subjects were informed that they would go through several steps, but they did not know what these steps would involve, i.e. when they were informed about step 1, they did not know what would happen in step 2 and so on.

Step 1, risk preferences: In step 1 we applied a method for eliciting risk preferences similar to Dohmen and Falk's (2006), which is a simple version of

---

[15] Also Dohmen and Falk (2006), who used exactly the same work task, found that monetary incentives significantly affected performance. The level of effort costs may be task specific, but Bruggen and Strobel (2007) find insignificant differences between chosen effort costs (using a predetermined cost function) and real effort costs when the task is multiplying numbers. Their study shows that multiplying numbers is an appropriate way to operationalize effort in experiments.

Holt and Laury (2002). Upon arrival the subjects were seated at a computer lot and given a table with 12 rows. For each row the subjects were asked to decide whether they preferred a lottery or a safe alternative. The lottery was a fifty-fifty probability of NOK 200 or zero, and was the same for all rows. The safe alternative was NOK 15 in row one, increasing with NOK 15 for each row.[16] By examining the shifting point from the lottery to the safe option, we get information on the subjects' risk attitudes. With the chosen value of the safe option, a risk neutral participant with monotonous risk preferences would choose the lottery for the first six situations and then switch to the safe option for the remaining situations.

Step 2, ability revelation: In step 2 of the experiment, subjects revealed their ability levels by multiplying one- and two-digit numbers for a period of five minutes. They were paid by a piece rate scheme giving NOK 5 per correct answer. The problems were given on the computer screen and the subject typed the answer to the problem using the keyboard. A message appeared on the screen telling the subject whether the answer was correct or not. After the five minute work period they received either the grade A, B or C, depending on how well they did compared to the others, and they were told that 1/3 received grade A, 1/3 grade B, and 1/3 grade C. [17] An IQ test or a math quiz without performance pay could have given a more neutral measure of ability. On the other hand, we acquired a good measure of their ability - or willingness - to exert effort on the kind of tasks in which they were going to compete.

---

[16] In the lottery drawing we used Holt and Laury's procedure. One of the 12 rows was drawn, and the participants were either paid zero, 200 or the row's safe alternative (if that was chosen by the participant). Drawings and payments were made at the end of the experiment, i.e. after step 5 (see below).

[17] By aggregating into only three grades, we use less information than we could have used. However, we achieve a simple and (for students) interpretable measure of ability. Moreover, we wanted to compare heterogeneous and homogenous pairs, and aggregation was necessary in order to get a sufficient number of homogenous pairs.

Step 3, bargaining: In step 3, subjects were told that they in the next step were going to compete against another subject in doing similar kinds of head calculations for five minutes. The subjects were then asked to split NOK 200 by deciding a winner prize and a loser prize (w_1, w_2) prior to this competition. Subjects were randomly picked to either propose prizes (*proposer*) or choose to accept or reject prize proposals (*responder*). Accept yielded the proposed solution, but if an offer was rejected, then prizes were set to (150,0).[18] We imposed uncertainty by telling the subjects that a random variable, called a bonus, would be drawn after the competition (tournament) and added to the subjects' number of correct answers. We imposed two levels of uncertainty: the random bonus either had uniform distribution between -3 and 3 ("low uncertainty"), or uniform distribution between -10 and 10 ("high uncertainty"). Ability-levels (for proposer and responder) and the uncertainty-level were common knowledge when they bargained.

Each subject participated in four rounds of bargaining, where they met new opponents each round. They were told that one out of the four rounds would be picked at random to determine the prizes for the oncoming tournament. There were two bargaining rounds were subjects where told that the random bonus was distributed between -3 and 3, and two rounds where they were told that the bonus was distributed between -10 and 10. After each round of bargaining, subjects were informed about the outcome of the bargaining.

Step 4, tournament. Subjects went through a new five-minute work period multiplying one- and two-digit numbers. They knew the grade of their opponent (as well as their own), the size of the prizes and level of

---

[18] The rejection prizes reflect the cost of bargaining break down (lower total surplus), and the idea that a principal in general would like a higher prize spread than the agents.

uncertainty. The sequence of problems were the same for all subjects and in case of a tie, randomization determined the winner. After the work period, winners and losers were revealed together with the number of correct answers and the random individual bonus (luck component).

Step 5, questionnaire. We gathered questionnaire data on gender, age and personality. Personality was measured by the Big-Five scale used by psychologists, which measures the degree of Openness, Conscientiousness, Extraversion, Agreeableness and Neuroticism.[19]

4 Hypothesis

Assume that the subjects believe their ability assignment. In equilibrium, the responder then accepts the proposer's offer in the bargaining game outlined above. In games where the best subject proposes (A to B, A to C or B to C), she has all the bargaining power since the low-ability subject has nothing to earn from rejecting the offer. In games where the low-ability subject proposes (B to A, C to A or C to B), the high-ability subject has some bargaining power, since she can gain from refusing an offer with sufficiently low prize spread. The low-ability subject will offer the lowest prize spread that the high-ability subject is expected to accept. Hence, the qualitative comparative statics results on the optimal spread for the high-ability agent applies for the bargaining solution also when the low ability subject proposes. The model thus predicts the following outcomes from our experiment:

H1: *Among heterogenous pairs, prize spread is higher when the random bonus has distribution $U(-3,3)$, than when it has distribution $U(-10,10)$.*

---

[19] The Big-Five questionnaire measures personality traits by asking subjects how they assess themselves. We used a 20 item version of the questionnaire. The subjects indicate their assessments on a seven-point scale for each item.

H2: *Prize spread increases with ability-difference.*

H3: *Effort increases with ability-difference.*

In our model effort equals number of correct answers, while output is number of correct answers plus the randomly chosen bonus.

5 Results and analysis

In this section we present the main results. Table 1 displays summary statistics on prize spread by pair composition and level of uncertainty.

Table 1. Average prize spread by pair composition and uncertainty level

	AC-pairs	AB-pairs	BC-pairs	Homogenous pairs	Overall
Avg. prize spread low uncertainty	121,9	98,3	105,7	55,0	99,7
	(33.9)	(48.1)	(34.5)	(35.9)	(44.6)
Avg. prize spread high uncertainty	102,3	93,7	97,7	62,2	91,9
	(39.9)	(53.5)	(48.4)	(64.2)	(51.8)
Avg. prize spread total	112,1	96,0	101,7	58,6	95,8
	(38.0)	(50.5)	(41.9)	(52.5)	(48.4)
Observations (low/high/total)	30/30/60	30/30/60	30/30/60	18/18/36	216

Notes : The table presents average prize spreads (standard deviation in parentheses) by pair composition and uncertainty level. AC-pairs refers to pairs of subjects graded A and C in step 2 of the experiment (likewise for AB-pairs and BC-pairs). Homogeneous pairs consists of pairs of equal gradingin step 2. Low [high] uncertainty refers to the random bonus distribution U(-10,10) [U(-3,3)].

An AB-pair consists of a subject graded A who is bargaining against a subject graded B. A bargaining solution from an AB-pair is either the outcome from an A's offer to a B, or a B's offer to an A. The same goes for AC-pairs and BC-pairs. Homogeneous pairs consist of bargaining solutions from A vs. A, B vs. B or C vs. C. "Low uncertainty" refers to random bonus distribution $U(-3,3)$, while "High uncertainty" refers to random bonus distribution U(-10,10).

Two tendencies are shown in Table 1: First, we observe that prize spread decreases with uncertainty-level. Except for the homogenous pairs, prize spread is lower under high uncertainty than under low uncertainty. This supports H1. Second, we see that prize spread increases quite strongly with ability-difference. It is lowest for the homogenous pairs and highest for the AC pairs. This also corresponds with the prediction of the model and seems to support H2 above.

Let us examine H1 more closely. First, we report on a t-test of H1, dropping homogenous pairs from the sample (since H1 does not apply for homogenous pairs). We test the hypothesis that prize spread is the same under both low- and high uncertainty against the one-tailed alternative that prize spread is *higher* under low uncertainty than under high uncertainty. A two sample t-test makes us reject the null-hypothesis of equal prize spread; prize spread is significantly higher under low uncertainty ($t(178) = 1.65$, $p = 0.05$, one-tailed). When we run a regression, controlling for risk aversion, pair composition (heterogeneity) and gender, we get the same picture, see Table 2. The coefficient on uncertainty-level ("high") is statistically significant within a 90 % confidence interval ($p = 0.09$). Controlling for risk preferences, pair composition and gender, the regression shows a decrease in prize spread of NOK 11.0 (1.4 Euro) when going from low uncertainty to high uncertainty. Importantly, we see that risk preferences cannot explain prize spread. It can also be shown that interaction variables on risk preferences and uncertainty-level are highly insignificant. This may seem surprising, but the majority of subjects are risk neutral or close to risk neutral over the relatively low stakes offered here. We thus establish our first main result from the experiment:

Result 1 *Controlling for risk preferences, prize spread is higher under low uncertainty than under high uncertainty.*

Result 1 supports H1. An alternative explanation for result 1 is that subjects have some kind of fairness concerns: High prize spread is alright if effort is important, but not if luck is important (see Cappelen et al., 2007, for an interesting experiment on the relationship between effort and distributive justice). Our design makes it impossible to exclude this explanation, but if fairness concerns are important, one would expect that gender and/or personality have an impact on prize spread per se, and on the relationship between prize spread and uncertainty. However, we find no significant effects on prize spread from personality and gender.[20] But we cannot conclude that fairness concerns do not play a role, and future research should thus incorporate fairness concerns (e.g. ala Fehr and Schmidt, 1999) in the model we present and control explicitly for it in the experiment.

Let us now consider H2: Prize spread increases with ability-difference. Table 2 indicates that there is a positive relationship between prize spread and ability-difference among heterogenous pairs, but since the corollary also applies when k_j is reduced from $k_j = k_i$ to $k_j < k_i$, we must also include the homogenous pairs in the sample. We first report on t-tests on the relationship between prize spread and each pair composition. Let $s(h)$, $h = AC$, $AB, CB, HOMO$ denote prize spread as a function of ability-difference. The tests support that $s(AC) > s(AB) = s(BC) > s(HOMO)$. From Table 3, we see that all tests are significant within a 90 % confidence interval except for $s(BC)$ vs. $s(AB)$, as predicted. We can thus state

Result 2 *Prize spread increases with ability-difference.*

Result 2 supports H2.[21] As predicted by the model, Result 2 should

---

[20] Several studies show that social preferences are stronger among women (see Croson and Gneezy, 2004, for a survey), and concerns for distributive justice have been shown to be correlated with personality traits derived from the Big-Five personality test (see Skarlicki et al. 1999).

[21] It can be shown that Results 1 and 2 hold when we control for who is proposer and who is responder. In particular, we find the same results when we examine the high-ability subjects' proposals.

also imply that effort increases with ability-difference. This leads us to H3. Table 4 displays a robust regression where effort, i.e. number of right answers (random bonus excluded) is the dependent variable. We see that prize spread has a significantly positive effect on effort ($p = 0.017$). For a NOK 1 increase in prize spread, the number of correct answers increases with 0.034. This may seem like a small effect, but it means that an increase from zero prize spread to max prize spread of NOK 200 increases the number of correct answers with 6.8. We thus have

Result 3 *Effort increases with prize spread.*

Results 2 and 3 support H3: Higher ability-difference increases prize spread, which in turn increases effort. But note from Table 4 that when we *control for prize spread*, ability-difference has a negative effect on effort. This fits with other findings in the literature (starting with Bull et al.,1987) and supports our model. From the IC constraints, we see that for a *given prize spread*, effort decreases with ability-difference. This result is well known and traces back to Lazear and Rosen (1981).

6 Conclusion

Tournament theory provides us with a tool for analyzing situations where agents compete for prizes, such as wage raises, promotions, championships or research funds. Often the prizes are set by non-participating parties, and so the literature has extensively analyzed tournaments where a principal determines rules and prize spread. However, sometimes the contestants themselves determine reward structure. In self-managed teams or democratic organizations, members or employees may vote over strategic decisions that involve rules for internal resource allocation. And labor-unions may affect wage dispersion under decentralized bargaining. Hence, in this paper we analyze and experimentally test a tournament model where heterogenous agents determine prize spread.

We find some results particularly interesting. First, our corner solutions elucidate empirical puzzles on firm characteristics and wage structure, since marginal differences in heterogeneity, uncertainty and size of the prize pool can significantly affect prize spread. The corner solutions also elucidate why it is often necessary for a principal to set the spread: With sufficiently low heterogeneity effort suffers dramatically if agents set the spread.

Second, our theoretical result on the positive relationship between heterogeneity and prize spread, supported by the experiment, challenges the idea that heterogenous agents should not participate in the same contest. If the agents determine prize spread, higher ability-difference triggers higher prize spread, resulting in higher effort.

Third, our model shows that if agents set prize spread in an asymmetric tournament, then we can expect a negative relationship between uncertainty and prize spread. This result is supported experimentally and has important empirical implications. It implies that an observed negative relationship between prize spread and uncertainty does not have to be explained by risk aversion or fairness concerns. Moreover, it suggests that the relationship between uncertainty and wage structure in an industry is affected by the bargaining power of employees: If the principal determines wage spread, higher uncertainty leads to higher wage spread. But if the agents determine wage spread, higher uncertainty leads to lower wage spread. Hence, bargaining regime and the degree of unionization should affect the relationship between uncertainty and wage spread.

Even if there exist prize-setting regimes that are similar to tournaments where the contestants themselves set prizes, it will often be done in cooperation or by bargaining with a non-participating principal. While we have addressed a tournament where the principal only determines the prize pool, future research should relax the constraint on the total prize, and investigate richer bargaining environments that include bargaining between agents and principal. Future work should also explore sorting, risk aversion and social preferences such as fairness concerns within the setting presented here.

Appendix

Proof of Proposition 2

Consider the marginal value $W'(w_2)$. For the given cost functions (see (6) and (7)) the effort difference is $e \equiv e_i - e_j = \frac{1}{K} e_j$, where $K = \left((\frac{k_j}{k_i})^{\frac{1}{n-1}} - 1 \right)^{-1}$. From the IC constraint $(R - 2w_2)g(e) = C_j'(e_j)$ we have

$$(R - 2w_2)g'(e)\frac{\partial e}{\partial e_j}\frac{\partial e_j}{\partial w_2} - C_j''(e_j)\frac{\partial e_j}{\partial w_2} = 2g(e)$$

Substituting from this and the IC constraint into (5), and using $e_j = Ke$ we then get, after some algebra:

$$W'(w_2) = 1 - 2G(e) - K\frac{2g(e)e}{\frac{g'(e)}{g(e)}e - (n-1)} \equiv \hat{F}(e), \qquad e = e(w_2) \tag{9}$$

At an interior optimum we will have $W'(w_2) = 0$, and the optimal effort difference e given by $\hat{F}(e) = 0$. The second-order condition for an optimum requires $W''(w_2) = \hat{F}'(e)e'(w_2) \leq 0$. From the IC constraints and (7) we can see that $e'(w_2) < 0$ when $g'(e) \leq 0$ for $e \geq 0$, which we will assume to be the case. The SOC for an optimum thus requires $\hat{F}'(e) \geq 0$. Note that $e = 0$ is always a solution to $\hat{F}(e) = 0$.

For noise of the form (8), denote the CDF of $\varepsilon_j - \varepsilon_i$ by $\Gamma(d) = \Pr(\varepsilon_j - \varepsilon_i < d)$, with density $\gamma(d) = \Gamma'(d)$. Then we have

$$G(e) = \Pr(z_j - z_i < e) = \Pr(\varepsilon_j - \varepsilon_i < \frac{e}{\sigma}) = \Gamma(\frac{e}{\sigma})$$

and $g(e) = G'(e) = \gamma(\frac{e}{\sigma})\frac{1}{\sigma}$. By defining $d = \frac{e}{\sigma}$ we have $g(e)e = \gamma(d)d$ and $\frac{g'(e)}{g(e)}e = \frac{\gamma'(d)}{\gamma(d)}d$, and hence (9) can be written as

$$W'(w_2) = F(d) \equiv 1 - 2\Gamma(d) - K\frac{2\gamma(d)d}{\frac{\gamma'(d)}{\gamma(d)}d - (n-1)}, \quad d = \frac{e(w_2)}{\sigma} \tag{10}$$

We see that at an interior optimum the optimal effort difference e would be given by $e = \sigma d$, where d is a solution to $F(d) = 0$. The SOC then requires $F'(d) \geq 0$. The other possibilities are corner solutions; either $w_2 = 0$ or $w_2 = R/2$.

For $w_2 = R/2$ and thus $e = e(w_2) = 0$, we see that $W'(w_2) = F(0) = 0$. For this to be a maximum, the SOC requires $F'(0) \geq 0$. It turns out (see below) that this condition is satisfied iff $K \geq n-1$, i.e. iff the degree of heterogeneity is 'small' ($\frac{k_j}{k_i} \leq \left(\frac{n}{n-1}\right)^{n-1}$). In such a case, minimal effort and spread ($e = d = 0$ and $w_2 = R/2$) is then a local maximum.[22] Moreover, this maximum is also a global one if $F(d) > 0$ for $0 < d < d_m = e_{\max}/\sigma$, where $e_{\max}$ is the largest feasible effort spread for the given R, i.e. the spread corresponding to $w_2 = 0$. We now show that this is indeed the case if R is sufficiently small and/or σ is sufficiently large.

By straightforward calculations we find

$$F'(0) = 2\gamma(0)\left(-1 + K/(n-1)\right) \tag{11}$$

Hence $F'(0) > 0$ iff $K > n-1$. For $K > n-1$, i.e. low heterogeneity ($\frac{k_j}{k_i} < \left(\frac{n}{n-1}\right)^{n-1}$), we then have by continuity $F(d) > 0$ for $0 < d < d_1$, some $d_1 > 0$.

Let $e_{\max}$ be the effort spread corresponding to $w_2 = 0$; it is from the IC constraint and (6)-(7) given by

$$Rg(e_{\max}) = nk_j(Ke_{\max})^{n-1} \equiv k(e_{\max})^{n-1} \tag{12}$$

where $k = nk_j(K)^{n-1}$ is defined by the identity. Since $g(e) = \gamma(\frac{e}{\sigma})\frac{1}{\sigma}$ we

---

[22] More precisely, it is a local maximum if strict inequality $K > n-1$ and thus $F'(0) > 0$ hold.

then have $e_{\max} = \sigma d_m$, where d_m is given by

$$(R/\sigma^n)\gamma(d_m) = k(d_m)^{n-1}, \qquad (d_m = e_{\max}/\sigma) \tag{13}$$

We see that $d_m \to 0$ as $R/\sigma^n \to 0$, hence there is $r_1 > 0$ such that $d_m < d_1$ for $R/\sigma^n < r_1$, where d_1 was defined in the paragraph after (11). It thus follows that for $R/\sigma^n < r_1$ we have $F(d) > 0$ for $0 < d < d_m$, and hence $W'(w_2) > 0$ for $0 < w_2 < R/2$. The minimal spread $w_2 = R/2$ (and $e = 0$) is thus optimal here.

By a similar reasoning we can also show that for small R and/or large σ it is optimal to induce maximal effort spread when the degree of heterogeneity is 'large', i.e. when $K < n-1$ and thus $F'(0) < 0$. This completes the proof.

Proof of Proposition 3

Let $\bar{d}$ be the upper end point of the support for $\Gamma(.)$ (so $\Gamma(\bar{d}) = 1$), and consider $F(d, K)$ defined in (10). By straightforward calculations one can show that for normally or uniformly distributed noise terms the following conditions are satisfied:[23]

(c1) $F(d, K) < 0$ for all $d \in (0, \bar{d})$ when $K \leq n-1$

(c2) $F(d, K) \gtreqless 0$ as $d \lesseqgtr d_0$, $d \in (0, \bar{d})$, some $d_0 \in (0, \bar{d})$; when $K > n-1$

(c3) $\gamma'(d) < 0$ for $d \in (0, \bar{d})$, and $\gamma(d)d \to 0$ as $d \to \bar{d}$.

Let D be the class of distributions that satisfy (c1-c3). We show that the statements in the proposition hold for this class.

(i) Consider first low heterogeneity; $K > n-1$. Let $d_m = d_m(R/\sigma^n)$ be defined by (13). We see that $d_m'() > 0$ and $d_m \to \bar{d}$ as $R/\sigma^n \to \infty$, and hence that there is $r_0 > 0$ such that $d_m \lesseqgtr d_0$ as $R/\sigma^n \lesseqgtr r_0$, where d_0

---

[23] For the uniform case $\varepsilon_l \sim U[0,1]$ we have $\Gamma(d) = \Pr(\varepsilon_j - \varepsilon_i < d) = 1 - \frac{1}{2}(1-d)^2$, and for the normal case with $\varepsilon_i - \varepsilon_j \sim N(0,1)$, one can use the fact that the density satisfies $\gamma'(d) = -d\gamma(d)$.

is the root defined in (c2). For $R/\sigma^n < r_0$ we thus have $F(d,K) > 0$ all $d \in (0, d_m)$ and therefore $W'(w_2) > 0$ all $w_2 \in (0, R/2)$. This implies that $w_2 = R/2$ (and thus $e = 0$) is optimal for $R/\sigma^n < r_0$. The optimal value is then $W(R/2) = R/2$.

For $R/\sigma^n > r_0$ we have $d_m > d_0$ and hence $F(d,K) \gtreqless 0$ as $d \lesseqgtr d_0$, $d \in (0, d_m)$. There is thus w_{20} such that $W'(w_2) \gtreqless 0$ as $w_2 \gtreqless w_{20}$, $w_2 \in (0, R/2)$. Hence either $w_2 = R/2$ $(e = 0)$ or $w_2 = 0$ $(e = e_{\max})$ is then optimal. For $w_2 = 0$ the value is

$$W_0 = G(e_{im} - e_{jm})R - C_i(e_{im}) \tag{14}$$

where e_{im}, e_{jm} are the agents' respective efforts when $w_2 = 0$, and thus $e_{im} - e_{jm} = e_{\max}$. Consider now how the value W_0 varies with R. Since efforts depend on prizes via $R - 2w_2$, we will have $\frac{\partial e_{lm}}{\partial R} = -\frac{1}{2}\frac{\partial e_l}{\partial w_2}$. By the same reasoning that led from (5) to (9) and (10) we then obtain

$$\begin{aligned}\frac{\partial}{\partial R}W_0 &= G(e_{im} - e_{jm}) + g(e_{im} - e_{jm})\left(-\frac{\partial e_{jm}}{\partial R}\right)R \\ &= -\frac{1}{2}\left(\hat{F}(e_m) - 1\right) = -\frac{1}{2}\left(F(d_m) - 1\right)\end{aligned}$$

where $e_m = e(0) = e_{\max}$ and $d_m = \frac{e_m}{\sigma}$. Comparing the values corresponding to $w_2 = R/2$ and $w_2 = 0$ we thus have

$$\tfrac{\partial}{\partial R}(W(R/2) - W_0) = \tfrac{1}{2} - \tfrac{\partial}{\partial R}W_0 = \tfrac{1}{2}F(d_m)$$

From property (c2) it then follows that the value difference is increasing for $R < r_0\sigma^n$ (where $d_m < d_0$) and decreasing for $R > r_0\sigma^n$. If the difference is negative for R sufficiently large, it then follows that there is $R_1 > 0$ such that $W(R/2) \gtreqless W_0$ as $R \lesseqgtr R_1$.

Consider W_0 given by (14); we have $W_0 = G(e_m)R - K_i(e_m)^n$, where

$K_i = k_i \left(1 - (\frac{k_i}{k_j})^{\frac{1}{n-1}}\right)^{-n}$, $e_m = e_{\max} = \sigma d_m$, and d_m is given by (13). Using (13) we then find, after some algebra:

$$\frac{W_0}{W(R/2)} = 2\left(\Gamma(d_m) - \frac{K_i}{k}\gamma(d_m)d_m\right) \tag{15}$$

Since $d_m \to \bar{d}$ and thus $\gamma(d_m)d_m \to 0$ as $R \to \infty$ (by (13) and (c3), respectively), we see that $W_0 > W(R/2)$ for R sufficiently large. Since moreover d_m and hence the ratio $\frac{W_0}{W(R/2)}$ depends on R and σ via R/σ^n, we see that there is indeed $r_1 > 0$ such that $W_0 > W(R/2)$ iff $R/\sigma^n > r_1$. This proves statement (i) in the proposition.

(ii) Consider next high heterogeneity; $K < n - 1$.

It follows from property (c1) that we then have $F(d, K) < 0$ for all $d \in (0, \bar{d})$, and therefore $W'(w_2) < 0$ for all $w_2 \in (0, R/2)$, for any (feasible) $R > 0$. Hence $w_2 = 0$ is always optimal in this case. This proves statement (ii).

Finally, consider statement (iii). We have $e_{\max} = \sigma d_m(R/\sigma^n)$, where $d_m(.)$ is defined by (13). Differentiation of (13) shows that $d_m'() > 0$, and yields, after some algebra

$$\frac{\partial}{\partial\sigma}e_{\max} = d_m(R/\sigma^n) - n(R/\sigma^n)d_m'(R/\sigma^n) = d_m\left[1 - \frac{n\gamma(d_m)}{\gamma(d_m)(n-1) - \gamma'(d_m)d_m}\right] \tag{16}$$

For $\sigma \to 0$ we have $R/\sigma^n \to \infty$, and by (13) and (c3) $d_m \to \bar{d}$, implying $\left(\frac{\partial}{\partial\sigma}e_{\max}\right)/d_m \to 1 > 0$. For $\sigma \to \infty$ we have $R/\sigma^n \to 0$, and by (13) and (c3) $d_m \to 0$, implying $\left(\frac{\partial}{\partial\sigma}e_{\max}\right)/d_m \to 1 - \frac{n\gamma(0)}{\gamma(0)(n-1)+0} < 0$. This proves statement (iii) and completes the proof.

Tables and figures

Table 2: Prize spread and uncertainty

Dependent variabe: Prize spread	Coefficient	p-value
High uncertainty	-10.976*	0,09
AB-pair	-15.780*	-0,05
BC-pair	-10.935	0,14
Male	11.113	0,33
Risk averse pair	-3.364	0,68
Risk loving pair	-5.552	0,5
Non-monotonic pairs	-8.891	0,20
Constant	119.826***	<0.00
R-squared	0.052	
Sample size	180	

Notes: Robust OLS regression. Level of significance: * = 0.10, ** = 0.05, *** = 0.01. Homogenous pairs not included. The dummy "High uncertainty" is equal to one if uncertainty is high and zero if uncertainty is low. Low [high] uncertainty refers to the random bonus distribution U(-10,10) [U(-3,3)]. "Male" is the gender dummy, which equals one if the pair consists solely of men and zero if a woman is part of a pair. The reference group for pair composition is AC pairs. The dummy "AB-pair" equals one if the pair is an AB-pair and zero otherwise. The same goes for the "BC-pair" dummy.

For risk preferences we use the switching point in the lottery choices elicited in step 1 of the experiment, and categorize subjects' risk preferences in four different risk categories; Risk neutral, risk averse, risk loving and non-monotonic risk preferences. A risk neutral subject would switch from the lottery to the safe alternative after six lottery choices, risk averse subjects switch earlier while risk loving subjects switch later. Non-monotonic subjects have multiple switching points.

The dummy "Risk averse pair" equals one if at least one subject in a given pair is risk averse and no-one is risk loving, and zero if not. "Risk loving pair" equals one if at least one subject is risk loving and no-one is risk averse, and zero otherwise. Risk neutral pairs are the reference group, and consist of pairs where both subjects in a pair are risk neutral, or one is risk loving while the other is risk averse. As in Dohmen and Falk (2006) there were some subjects having non-monotonic risk preferences, making us include a "non-monotonic pairs" dummy equal to one if both subjects have multiple switching points in the lottery choice experiment.

Table 3: Prize spread differences

Average prize spread in:	Welch`s t-test
AC-pairs vs. AB-pairs*	p =.03
AC-pairs vs. BC-pairs*	p =.08
BC-pairs vs. AB-pairs**	p =.51
AC-pairs vs. Homogeneous pairs*	p <0.001
AB-pairs vs. Homogeneous pairs*	p <0.001
BC-pairs vs. Homogeneous pairs*	p <0.001

Notes : The tabel presents t-tests for differences in prize spread between pairs of different ability composition.
* : one-sided t-test. ** : two-sided t-test.

Table 4: Effort and ability-difference

Dependent variable: Effort	Coefficient	p-value
Prize spread	0.034**	0.017
AC	-5.267**	0.022
AB	-1.458	0.528
BC	-3.206	0.104
A	9.186***	p<0.001
C	-6.141***	p<0.001
Constant	10.981***	p<0.001
R-squared	0.510	
Sample size	108	

Notes: Robust OLS estimates. Level of significance: *=0.10, **=0.05, ***=0.01. Here the data are on individuals, not pairs, making the dummies AB, AC and BC equal one if the subject was part of the relevant pair. The reference group consists of subjects that were part of homogeneous pairs. The dummies A and C are ability-levels, with B as reference group.

Table 5: Effort data

Effort piece rate Ability level:	A	B	C	Effort tournament Ability level:	A	B	C
Mean effort	21,4	9,5	3,2	Mean effort	40,4	12,6	4,6
Std. Dev	4,76	3,38	1,62	Std. Dev	9,71	6,34	3,1
Median	20,5	9,5	3	Median	19,5	13	4
Min	16	4	1	Min	5	1	1
Max	33	17	6	Max	38	25	12
# obs.	36	36	36	# obs.	36	36	36

Notes: The tabel presents effort by ability level for the piece rate experiment and for the tournament experiment. Effort equals number of right answers in the 5 minute work period.

References

Boot, Allison. 1984. A public choice model of trade union behavior and membership. *Economic Journal* 94: 883-898.

Brüggen, Alexander and Martin Strobel. 2007. Real effort versus chosen effort in experiments. *Economics Letters* 96: 232–236.

Brunello, Giorgio. 1994. Incentives, bargaining and the prize structure. *The Scandinavian Journal of Economics* 4: 481-497.

Bull, C., A. Schotter and K. Weigelt. 1987. Tournaments and piece rates: an experimental study. *Journal of Political Economy*, 95: 1–33.

Cahuc, Pierre and Andre Zylberberg. 2004. *Labor Economics.* The MIT Press, Cambridge, Massachusetts.

Cappelen, Alexander, Astri D. Hole, Erik Ø. Sørensen and Bertil Tungrodden. 2007. The pluralism of fairness ideals: an experimental approach, *American Economic Review* 97: 818-827, 2007.

Carmichael, H. Lorne. 1983. The agent-agents problem: payment by relative output. *Journal of Labor Economics* 1: 50-65.

Croson, Rachel and Uri Gneezy. 2004. Gender Differences in Preferences. Mimeo. Rady School of Management.

Dahl, Christian M., Daniel Le Marie and Jakob R. Munch. 2009. Wage dispersion and decentralization of wage bargaining. Discussion Paper, University of Copenhagen.

Dijk, F. van , J. Sonnemans and F. van Winden. 2001. Incentive systems in a real effort experiment. *European Economic Review* 45: 187–214.

Dohmen, Thomas and Armin Falk. 2006. Performance pay and multidimensional sorting: productivity, preferences and gender. IZA discussion paper no. 2001.

Gibbons, R., R. Henderson, N. Repenning and J. Sterman. 2007. What Do Managers Do? Suggestive Evidence and Potential Theories About Building Relationships. Forthcoming in *Handbook of Organizational Economics* (R. Gibbons and J. Roberts, eds.), Princeton University Press.

Fehr, E. and K. Schmidt. 1999. A theory of fairness, competition, and cooperation. *Quarterly Journal of Economics* 114: 769–816.

Fischbacher, U. 1999. Z-tree. Zurich toolbox for readymade economic experiments. *Experimental Economics* 10: 171-178.

Green, Jerry R. and Nancy L. Stokey. 1983. A comparison of tournaments and contracts. *Journal of Political Economy* 91: 349-64.

Harbring, Christine and Bernd Irlenbusch. 2004. Incentives in tournaments with endogenous prize selection. *Journal of Institutional and Theoretical Economics* 161: 636-663.

Holmström, Bengt. 1982. Moral hazard in teams. *Bell Journal of Economics* 13: 324-40.

Holt, Charles A. and S. K. Laury. 2002. Risk aversion and incentive effects. *American Economic Review* 92: 1644-1655.

Jehn, K. A., G. B. Northcraft and M. A. Neale. 1999. Why differences make a difference: A field study of diversity, conflict and performance of workgroups. *Administrative Science Quarterly* 44: 741-63.

Konow, J. 2003. Which is the fairest one of all? A positive analysis of justice theories. *Journal of Economic Literature* 41: 1188-1239.

Kräkel, Matthias and Dirk Sliwka. 2004. Risk taking in asymmetric tournaments. *German Economic Review* 5: 103-116.

Lazear, Edward P. and Sherwin Rosen. 1981. Rank-order tournaments as optimum labor contracts. *Journal of Political Economy* 89: 841–864.

Mookherjee, Dilip. 1984. Optimal incentive schemes with many agents. *Review of Economic Studies* 51: 433-46.

Osterman, P. 1995. How common is workplace transformation and who adapts it? *Industry and Labor Relations Review* 47: 173-187.

Prendergast, Canice. 2002. Uncertainty and incentives. *Journal of Labor Economics*, 20: 115-137.

Riis, Christian. 2009. Efficient contests. Working paper. Norwegian School of Management.

Ross, A. 1948. *Trade Union Wage Policy*, Berkeley and Los Angeles, University of California Press.

Seip, Åsmund A. and Tor A. Stokke. 2003. Lokal lønnsdannelse og tvisteløsning. Erfaringer fra nordeuropeiske land (Local wage formation and dispute settlement. Experiences from North European countries). *Fafo-rapport 422.*

Seip, Åsmund A. 2002. Lokale lønnsforhandlinger i Staten. Lønn som personalpolitisk verktøy (Local wage bargaining in the government sector. Wages as a tool for HR policy). *Fafo-rapport 397.*

Skarlicki, D. P., R. Folger and P. Tesluk. 1999. Personality as a moderator in the relationship between fairness and retaliation. *Academy of Management Journal* 42: 100–108.

Sutter, Matthias. 2006. Endogenous versus exogenous allocation of prizes in teams - theory and experimental evidence. *Labour Economics* 13: 519-549.

Yeatts, Dale E. and Cloyd Hyten. 1997. *High-Performing Self-Managed Work Teams: A Comparison of Theory to Practice.* Thousand Oaks, CA: Sage.