

**An evaluation of the reserve price at a  
discriminatory sealed-bid auction**

by

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### **Abstract**

A nonparametric estimation method of the distribution of bidders' private values is used in order to derive an estimate of the lower bound for the optimal reserve price at a Norwegian first-price or discriminatory sealed-bid auction of mackerel catches. Although the theoretical model that is used is not an exact mapping of the data generating process, it is shown that the potential errors in using the model are one-sided. It is shown that a consistent underestimation of valuations lead to an underestimation of the optimal reserve price as well. Thus, the recommendation of an optimal reserve price is not too high. The analysis suggests that the reserve price should be raised by at least 14 percent. The theoretical derivation of the reserve price is based on large sample properties. In small samples, the possible inefficiencies that a raised reserve price may produce, can outweigh the gains.

# 1 Introduction

Empirical analyses of auctions draw on the theory of games of incomplete information. In the case of first-price, sealed-bid auctions, Riley and Samuelson [16] showed that there is a closed-form Bayes–Nash equilibrium solution of bidder’s strategies under the assumptions that buyers have independent and identically-distributed private values (symmetric IPV) and are risk neutral. The symmetric equilibrium strategy characterized the bid function as a monotonically increasing function of bidders’ private valuations of the object for sale. In this paper, we assume that the Riley and Samuelson model of independent private values is, as an approximation, a reasonable theoretical framework for an analysis of a carefully chosen sample of the Norwegian mackerel market. This makes it possible to analyse the auction market from the perspective of optimal mechanism design.

A central question is how the seller should design his auction format in order to garner the most revenues. We restrict our analysis to the auction format that is being used currently. Only improvements in the closed discriminatory auction are considered. At closed, discriminatory auctions, the only mechanism the seller has control over is the reserve price. Myerson [13] and Riley and Samuelson [16] derived an implicit relation from which the optimal reserve price can be obtained in the symmetric IPV model. Our purpose is to estimate the necessary elements of the formula in order to apply it on our real-world market.

In order to implement the optimal reserve price, we need information concerning the distribution of private values on which bidders base their bids. Typically, at auction markets, bids are observed, while private values and their distribution are unobserved. Advances in the structural econometrics of auction data have focused on recovering unobservable elements of bid strategies; Paarsch and Hong [15] provided a comprehensive presentation of the literature. The benefit of this novel approach is that it enables researchers to address policy questions regarding a given format and to compare the actual format with counterfactual formats.

We rely on the nonparametric two-step estimator of Guerre, Perrigne, and Vuong [5] for estimating the underlying distribution of private values implied by observed bids and, thus, to obtain an estimate for a lower bound on the optimal reserve price. The estimator is derived under quite restrictive as-

sumptions. From an empirical perspective, we need a sample of homogenous goods and a fixed set of potential bidders. From a theoretical perspective, the estimator is developed for the sale of one object within the independent private-values model. Our main contribution in this paper is to show that the estimator can also be used under more complicated auction formats and assumptions, given appropriate interpretations of results.

We acknowledge that our real-world market does not fit into the theoretical model without friction. Some elements of the auction format and the market—notably the simultaneous sales format, and, possibly, an endogenously determined number of participants—may give rise to an equilibrium solution that differs from the model. However, we shall argue that the potential error from our empirical model is one-sided. In particular, we show that the estimation procedure produces results that consistently underestimates true valuations; the procedure will not overestimate valuations. Relying on a result from the theory of stochastic orders, we then show that our estimate of the optimal reserve price will also consistently underestimate the true optimal reserve price. Therefore, our estimate will represent a lower bound on the optimal reserve price. Thus, one contribution of the paper is to show that relevant policy recommendations can be obtained by applying a seemingly restrictive model to a more complex real-world market.

An important paper analysing the optimal reserve price by applying a structural econometric approach is Paarsch [14]. He studied timber sales in British Columbia conducted by an open, ascending-price auction. His estimates of the optimal reserve price suggested that the current reserve prices used at the time should be substantially raised. The work seems to have had an impact as the government increased reserve prices at subsequent auctions.

The remainder of the paper is organized in the following way: First, we present the theoretical framework of our auction market. The bid function and optimal reserve price are presented. The important issue of identifying private values from observed bids is addressed as well in this section. Next, the assumptions of the theoretical model are discussed in detail in order to determine whether the model is appropriate for the real-world market. In section 4, we present the empirical strategy used for obtaining estimates of the relevant elements in the model. In section 5, we present and discuss our results. In the last section, we provide some concluding remarks.

## 2 The model

In this section, we introduce the notation and present the equilibrium bid function of a first-price, sealed-bid auction for which risk neutral bidders receive valuations independently from a symmetric distribution. This bidding function imposes structure on our auction data. The allocation rule is that the bidder with the highest bid above the reservation price  $r$  wins the object and pays his bid. Next, we present the optimal reserve price in this game. To investigate empirically the reserve price, we need to link observable bids to unobservable valuations. At the end of the section, we present the important identification result which our empirical estimation procedure rests on.

### 2.1 The bid function

Let  $\mathcal{N}$  be the number of potential bidders. Each bidder  $i$  has a valuation  $v_i$  that is independently drawn from an identical distribution with support  $[\underline{v}, \bar{v}]$ . Let  $V$  represent the random variable for which realization is denoted  $v$ . The population density function (pdf) of  $V$  is denoted  $f_V(v)$  with a corresponding cumulative distribution function (cdf)  $F_V(v)$ . In the language of games with incomplete information,  $F_V(v)$  represents the distribution of *types*. From a representative bidder's perspective, his bid maximizes expected profit. Let  $b = \beta(v)$  denote the common bid strategy which is assumed to be a strictly increasing and differentiable function of private value  $v$ . Under these assumptions,  $\beta(v)$  has a unique inverse,  $v = \beta^{-1}(b)$ . Conditional on winning, a bidder's profit is the difference between his valuation and his bid. The probability of winning is derived on the assumption that bid strategies are strictly increasing in valuations. Thus, the probability of winning is equal to the probability that all competitors have valuations below  $v$ :

$$\Pr(v_{-i} < v) = \prod_{-i} F_V(v) = F_V(v)^{\mathcal{N}-1},$$

where the notation  $v_{-i}$  denotes the vector of all valuations but  $v_i$ . The bidder's optimization problem is

$$\max_b \underbrace{(v - b)}_{\text{Profit if winner}} \times \underbrace{[F_V(\beta^{-1}(b))]^{\mathcal{N}-1}}_{\text{Probability of winning}}.$$

The maximization problem formulated above highlights that expected profit is the product of two factors that must be balanced in the equilibrium solution of the game. The higher the bid, the higher is the probability of winning, but at the same time, the lower is the profit should the player win the auction. The key to the identification problem in section 2.3 below, is given by the first-order condition for profit maximum (after substituting  $\beta(v)$  for  $b$ ):<sup>1</sup>

$$(v - \beta(v))(\mathcal{N} - 1) \frac{f_V(v)}{\beta'(v)} = F_V(v). \quad (1)$$

Let the seller set a minimum price he is willing to trade the object for, i.e., he sets a reserve price  $r$  which is not necessarily identical to his true valuation. In solving the maximization problem, we impose the boundary condition that a bidder with a valuation below the reserve price does not submit a bid, i.e.,  $\beta(v|v < r) = 0$ . We look for a symmetric Bayesian-Nash equilibrium. As shown by Riley and Samuelson [16], this ordinary differential equation has a closed form solution. The equilibrium bid function at this first-price, sealed bid auction with a known reserve price is:

$$b = \beta(v; r, \mathcal{N}, F_V) = v - \frac{1}{[F_V(v)]^{\mathcal{N}-1}} \int_r^v [F_V(u)]^{\mathcal{N}-1} du. \quad (2)$$

The intuitive interpretation of the bid function is that valuations are shaved by a factor that depends on the number of competitors ( $\mathcal{N} - 1$ ), the reserve price  $r$ , and the prior estimate of other bidders' valuations represented by  $F_V(v)$ . Note that bids are strictly increasing in  $\mathcal{N}$ . It can be shown that the optimal bid strategy is to bid the expectation of the second highest order statistic of private values, conditional on the bidder having the highest private value.

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<sup>1</sup>The first order condition is:

$$- [F_V(\beta^{-1}(b))]^{\mathcal{N}-1} + (v - b)(\mathcal{N} - 1) [F_V(\beta^{-1}(b))]^{\mathcal{N}-2} f_V(\beta^{-1}(b)) \frac{d\beta^{-1}(b)}{db} = 0$$

Since  $b = \beta(v)$ ,  $d\beta^{-1}(b)/db = 1/\beta'(v)$  and  $\beta^{-1}(b) = v$ , we can use these equalities and rearrange terms to obtain equation (1).

## 2.2 The optimal reserve price

In terms of optimal mechanism design, the only parameter the seller has control of in this model is the reserve price. Assume the good is worth  $v_0$  to the seller, but he sets a reserve price  $r$ . What level of  $r$  will enhance revenues most? The optimization problem for the seller is to maximize expected revenue. Note, first, that his revenue depends on whether the object is sold or not. Assume that the seller's individual valuation  $v_0$  is also drawn from  $F_V$ . His expected utility from retaining the object is  $v_0 [F_V(r)]^\mathcal{N}$  where  $[F_V(r)]^\mathcal{N}$  is the probability that all bidders have valuations below  $r$ . If the object is sold, then the expected revenue is equal to the expected highest bid. Denote the random variable of the highest bid by  $V_{\mathcal{N}:\mathcal{N}}$ , i.e.,  $V_{\mathcal{N}:\mathcal{N}}$  is the first order statistic of  $\mathcal{N}$  draws from the distribution  $F_V(v)$ . Since the winner has the highest valuation, the cumulative distribution function of  $V_{\mathcal{N}:\mathcal{N}}$  is  $[F_V(v)]^\mathcal{N}$  with associated probability density function  $\mathcal{N} [F_V(v)]^{\mathcal{N}-1} f_V(v)$ . The problem of maximizing expected revenue with respect to the reserve price then takes the form:

$$\max_r v_0 [F_V(r)]^\mathcal{N} + \int_r^{\bar{v}} \beta(v) \mathcal{N} [F_V(v)]^{\mathcal{N}-1} f_V(v) dv. \quad (3)$$

Substituting (2) into (3), and simplifying the expression by integration by parts, for the details see Hauge [8, chapter 2], we arrive at the somewhat easier problem:

$$\max_r v_0 [F_V(r)]^\mathcal{N} + \mathcal{N} \int_r^{\bar{v}} [v f_V(v) - 1 + F_V(v)] [F_V(v)]^{\mathcal{N}-1} dv.$$

The first term is the expected value to the seller if the product is unsold. The second term is the expected revenue obtained from selling the object to the highest bidder, at a price greater than  $r$ . The global maximum of the objective function must satisfy the first-order condition

$$v_0 \mathcal{N} [F_V(r)]^{\mathcal{N}-1} f_V(r) - \mathcal{N} [r f_V(r) - 1 + F_V(r)] [F_V(r)]^{\mathcal{N}-1} = 0. \quad (4)$$

Rearranging and simplifying equation (4) yields the result that the optimal

reserve price,  $r^*$ , is defined implicitly by

$$r^* = v_0 + \frac{1 - F_V(r^*)}{f_V(r^*)}. \quad (5)$$

Interestingly, the optimal reserve price is independent of the number of bidders and is strictly greater than the seller's reservation valuation  $v_0$ . A key assumption for this result is that valuations are private, and not correlated. Levin and Smith [11] analyzed the case with correlated valuations. Under specific rules governing how valuations are correlated, assuming exogenous entry and that the seller commits to not re-offer unsold objects, they find that the optimal reserve price converges to the seller's private value  $v_0$  as the number of bidders increases.

We note for later use that the last term on the right hand side of equation (5) is the inverse hazard rate of  $F_V$ . In terms of actually computing the optimal reserve price, we express the optimization problem as

$$r^* = \arg \max [(r - v_0)(1 - F_V(r))].$$

Two possibilities must be considered when raising the reserve price above the seller's own valuation. First, there is a risk that no one has a valuation above the reserve price, but there is at least one valuation in the interval  $[v_0, r]$ . In that case, the seller will incur a loss since no bids are submitted. Second, there is a chance that the reserve price is set in the interval between the two highest valuations. Recall that all bidders aim their bids at the conditional expected second highest valuation. Since all players submit their bids conditional on being the winner, they all take into account the possibility that the reserve price is above the second highest valuation. Consequently, bidders will shave their bids less than in the case of no reserve price since the strategy space is more narrow. The reserve price is most important when the number of bidders is low. If  $\mathcal{N}$  is large, then sheer competition is a good substitute for an optimal reserve price. But, notice from the bid shaving element in equation (2) that *all* bids for bidders with valuations above  $r$ , are pushed upwards under the sealed-bid format when a reserve price is introduced.

Auctions are typically a preferred sales mechanism when the seller is uncertain about the market price. Prospective buyers, who know their own

private value, have an informational advantage which they use to acquire the good at a price below their valuation. A consequence of introducing a reserve price above the seller's own valuation is that the auction format may no longer yield efficient outcomes. There is a probability that the bidder with the highest valuation above  $v_0$ , will not win the object. The seller—like any monopolist—finds it to his advantage to deviate from the Pareto optimal allocation that an efficient trade mechanism constitutes. Inefficiency is introduced in order to try to capture some of the informational rent the winner otherwise obtains.

### 2.3 Identification of valuations

In the sample of auctions we study, all bids and the reserve price are observed, while valuations and their underlying distribution are unobserved. The issue is whether the unobserved private values can be expressed in terms of variables that are either observed directly or can be estimated. Guerre, Perrigne and Vuong [5] showed that this is indeed the case.

Since bids are observed, we can estimate their distribution directly. Let  $G_B(b)$  be the distribution of observed bids with support  $[r, \beta(\bar{v})]$ . It can be shown that  $G_B(b)$  and the corresponding density function,  $g_B(b)$ , can be expressed in terms of unobservable elements of the first-order condition given by (1). We have that

$$G_B(b) = \frac{[F_V(v) - F_V(r)]}{[1 - F_V(r)]} \quad (6)$$

and

$$g_B(b) = \frac{f_V(v)}{\beta'(v)} \frac{1}{[1 - F_V(r)]}. \quad (7)$$

The details of arriving at the above probability functions are in appendix A.1. Notice that  $G_B$  and  $g_B$  are conditional probability functions while  $F_V$  and  $f_V$  are unconditional probability functions. Using equation (6), substitute  $[G_B(b)[1 - F_V(r)] + F_V(r)]$  into equation (1) for  $F_V(v)$ . Using equation (7), substitute  $\{g_B(b)[1 - F_V(r)]\}$  for  $f_V(v)/\beta'(v)$ . After some algebraic manipulations, we get

$$v = b + \frac{1}{\mathcal{N} - 1} \left\{ \frac{G_B(b)}{g_B(b)} + \frac{F_V(r)}{g_B(b)[1 - F_V(r)]} \right\}. \quad (8)$$

We now have an expression that links the unobservable private values  $v$  to variables that can be estimated from the observable bids. Equation (8) forms the basis for the estimator used in section 4.

### **3 The market**

Having established that there is a theoretical framework for interpreting our auction data, we discuss how well the market under analysis fits the maintained assumptions of the theoretical model. For a sound empirical analysis, we need to demonstrate that our theoretical model reasonably approximates the real-world auctions we study. We employ a structural empirical approach for our analysis of these auctions. A structural approach enables us to recover the unobservable objects in which we are interested. A motivating factor for the development of structural analysis of auction data was to enable researchers to address policy questions like: What is the best auction design in a given market? This is in contrast to the reduced-form approach in which standard hypothesis testing allows applied researchers to consider theory. The structural approach is a powerful one, but it hinges critically on the assumption that our model correctly characterizes the specific attributes of the world. Forcing our data into an incorrect model will produce dubious results. Thus, we discuss in this section how our market relates to the assumptions of the theoretical model.

Our strategy for ensuring consistency between data and the structural estimation of the theoretical model is based on two observations. First, we sampled auctions that most closely represent the single-object symmetric IPV model. Second, any remaining elements of our sample of auctions that deviate from the theoretical model actually reinforce our conclusions on the optimal reserve price.

The symmetric IPV model is based on one single object for sale. Bidders are symmetric and have independent and private valuations. Models with more complex information structures quickly become inherently difficult to analyse within the structural empirical approach. Consequently, the bulk of empirical work has been devoted to the symmetric IPV model.

### 3.1 The auction format

The auction house is NSS<sup>2</sup> located in Bergen, Norway. The auction is termed a “distance auction” where the seller reports his catch as soon as the catch is on board (the vessel is still at sea). The seller sets a preferred geographical sector where he prefers to deliver his catch. Then the auction is conducted, and the seller is told the location where his catch is to be delivered. The set of buyers vary depending on the deliverance sector since only the buyers located inside this sector are eligible to bid unconditionally. In order to have a well-defined market, we focus on one of these sectors where we can identify a stable set of potential buyers.

Bidders submit their bids via the internet. Bidders receive descriptions of the offered catches and at a scheduled time submit their sealed bids on the catches they want. The auction closes after an hour. There is no bundling of catches, bidders are free to bid on any they want. Catches vary in the total tons of fish they contain, but the fish are rather homogenous. Bids are stated as NOK per kilo, thus, ensuring a valuation measure independent of catch size. Some catches may consist of two or more lots if the catch can be divided into distinct weight classes or different species. In that case, bids are placed on each lot, and the average (over lots) highest bid determines the winner. This feature makes it easier to sample homogenous lots.

The reserve price is fixed over the entire season. If a catch goes unsold, it can be offered at the next auction with the same reserve price. Sellers have the option of setting individual reserve prices as long as they exceed the standard reserve price. If a catch with an individual reserve price is not sold, it can be offered at the next auction, but now with the standard reserve price. In one sense, the option of setting individual reserve prices gives sellers the opportunity to improve on the format, but it is rarely used in practice. One reason for this may be that the individual reserve price is not credible.

#### 3.1.1 The capacity constraint

Catches are sold *simultaneously*, and not sequentially. The combination of a simultaneous and sealed-bid format is potentially troublesome since capacity constraints are relevant in many markets. In our market, buyers

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<sup>2</sup>NSS is an abbreviation of *Norges Sildesalgslag*.

have short-term capacity constraints in processing or freezing of the raw material. A limited time is available to properly handle the fish before the product begins to deteriorate. Normally, individual bidders do not have the capacity to handle all of the supply at multi unit auctions. In closed auctions, if bidders have capacity constraints, the risk of winning too many objects is prevalent if players can bid on all objects.

For the auctioneer it is important that catches attract as many bids as possible since bids are strictly increasing in the number of participants. An elegant part of the auction mechanism in use, ensuring maximum participation, is that bidders can state capacity constraints as part of their bid vector. This works as follows. If a bidder has the highest bid on two catches of, say, 40 and 60 tons, and he sets a capacity limit equal to 70 tons, he will only be allocated one of them, and the remaining catch will go to the bidder with the second highest bid.<sup>3</sup> The option of declaring a capacity limit, makes it possible for bidders to bid independently on all objects. In particular, it frees them from strategic considerations concerning coordination of their bids. In the absence of such an option, bidders would have to reflect on what objects to bid on. They would prefer those where competition is weak. The maximum limit option ensures that competition in principle is equal for catches offered to the same delivery sector.

### 3.1.2 Priority of bids

Another characteristic of the auction rules is that bidders may give priorities to their bids. If the bidder in the example above, gives the 40 tons lot higher priority than the 60 tons lot, this determines that he will be allocated the smaller lot should he win both auctions. One reason for the priority rule, is to give buyers an opportunity to optimize their bundles. Given the nature of supply, most notably the different delivery sectors of individual sellers, bundling catches before offering them on the market is not practical.<sup>4</sup> But buyers will have preferences over bundles. Since catches differ in size, not all bundle combinations are equally attractive. Winning one large and one small lot may, for a given bidder at a given time, be better than winning two

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<sup>3</sup>If the second highest bidder also has a binding capacity constraint, the catch will go to the third highest bidder, etc.

<sup>4</sup>Chakraborty [2] analyzes whether auctioneers should bundle different objects before selling them. Under the standard auction formats, he finds that when the number of bidders is above a critical level, the seller prefers unbundled sales.

small or two large lots. Bidders will probably, in general, prefer to obtain total quantities as close to their capacities as possible.

In addition, the priority option gives structure to the potentially complex program of actually allocating the objects after the end of the bidding process. With no priorities, some discretion is left to the auctioneer with respect to the order of solving for capacity constraints. For an analysis of the strategic aspects of priorities under this auction format, see Hauge [8, chapter 8].

### 3.2 The effect of multi-object sales

Since bid strategies at single-object and multi-object auctions normally differ, we have to take a closer look at possible effects that multi-object sales in our mechanism might have on equilibrium bidding compared to the solution of the single-object sale.

Under a simultaneous-*independent* format, a classification of Weber [20], the sale of one object does not depend on the outcome of other sales. In our case, the highest bidder on any object normally wins the object. The only modification to the simultaneous-independent format is that, if capacity constraints are binding, the allocation of objects will, to some extent, be interdependent. Recall that the expected winning bid at the standard first-price, sealed-bid auction is the conditional expected second-highest private value. If a bidder knows that, on a given lot, the high bidder will not take the lot due to a capacity constraint, then he would condition his bid on this information and bid an amount equal to the conditional expected third-highest private value. But bidders have no information that makes it possible to predict reliably such occurrences. It is likely, however, that bidders expect capacity constraints to play a role at some auctions, and, consequently, bid less aggressively.

Below, we establish that some important elements of standard multi-unit auctions are likely to be absent from our mechanism.

**Constant marginal values.** Standard models of multi-unit auctions are formulated under the assumption that the marginal value of each acquired unit is decreasing; see Krishna [9, section 12.1]. For example, suppose a bidder that demands three units submits a bid vector  $(8, 7, 5)$ , meaning that he will pay 8 for one unit,  $8+7$  for two units, and  $8+7+5$  for three units. This

gives rise to the so-called demand reduction effect: When bidders have multi-unit demand, and the marginal value of units decreases, then bid shaving increases for each additional demanded unit; see, for instance, Milgrom [12, p. 258].

We consider the assumption of declining marginal values inappropriate in our market since bidders can explicitly state capacity constraints. Plants have a fixed capacity per day, notably cooling or freezing capacities, that cannot be adjusted in the short run. As long as they are operating within their capacity, marginal costs can reasonably be modelled as being constant. Thus, at a given auction, the marginal value of obtaining raw materials to be used for output in competitive markets is also constant. In other words, buyers have a flat demand within their capacity limit.

The driving factor that explains differences in marginal values for a given bidder over time, is the fluctuations in available capacity. When a buyer is replete with raw material, then his marginal value of obtaining more units drops significantly, perhaps close to zero.

Similarly, differences between different buyers' marginal values at a given time may also be explained by the short-term variation in their available capacities. Thus, the theoretical construct—that bidders draw valuations from the same distribution—seems especially appropriate in this market. The randomness of valuations—or the types—is explained by the fluctuations in capacities that give rise to variance in marginal values. To be precise on this essential point; we assume marginal values vary over time for a given bidder, and between types at a given time, but that a single distribution function of valuations captures this randomness.

The acceptance of the present auction format among buyers supports our maintained hypothesis that marginal values are constant when operating below the stated capacity constraints. Under the auction format, there is no direct way of formulating bids that reveal declining marginal values. Bids are independent; one “risks” winning *any* object one bids on, but due to the capacity constraint option, one does not risk winning *all* objects one bids on.

**Increased bid shaving.** Will bid shaving increase under a multi-object, simultaneous format? We start the discussion by introducing the notion of residual demand. The residual demand curve facing a bidder, is equal to

the total supply less the sum of the quantities demanded by other bidders given that this difference is positive, otherwise the residual demand is zero. In our market, residual demand is almost always zero for all bidders. On average, demand exceeded supply by a factor of 4.33 in this market in the 2003–4 season.

In general, however, it is not the case that every buyer individually can take away all supply. At several auctions in the dataset, it takes the aggregate demand from two or three buyers to “consume” the entire supply. Since each individual multi-unit demand in these cases is less than the total supply, bid shaving increases compared to the case of single-unit auctions.

The equilibrium price at an auction market will—as in the case of traditional demand-supply analysis—be determined by the marginal buyer; i.e., the buyer with a marginal value equal to the market clearing price. Consider first the case of single-unit auctions. At a first-price auction, the expected second-highest private value will determine the market price. In the case of multi-unit sales with unit demand, this reasoning carries directly over. Suppose there are four objects for sale, and each bidder only wants one unit. Then the equilibrium strategy is to bid equal to some conditional expectation of the fourth highest valuation. If  $\mathcal{N} > 4$ , then the equilibrium bid for bidder  $i$  is:  $\beta_i(v_i) = \mathcal{E} [V_{(\mathcal{N}-3:\mathcal{N})} | v_i \geq v_{(\mathcal{N}-3:\mathcal{N})}]$ . Under the assumption of single-unit demand, bids are, consequently, lower and bid shaving is larger at multi-unit than at single-unit auctions. The driving mechanism for this result is that buyers with fulfilled demand drop out and the competition for remaining objects is reduced. Under rational expectations, bidders will anticipate this effect and all bids at the auction will be adjusted downwards.

In our case, however, we have multi-unit demand; most bidders want more than one catch. This increases demand and competition compared with single-unit demand. The consequence is that bid shaving is somewhere between single-object and multi-object auctions with unit demand. To see this, consider the case of four objects for sale where each bidder wants two units. Assume the same number of participants as above in order to compare the prices obtained. In this case, the market price equals  $\mathcal{E} [V_{(\mathcal{N}-1:\mathcal{N})}]$ , which is strictly higher than  $\mathcal{E} [V_{(\mathcal{N}-3:\mathcal{N})}]$ . The conclusion is that since we have multi-unit demand, the bids will deviate from the single object case to a lesser degree than under multi-object, unit-demand auctions.

Recall the two essential elements of the auction format; it is a discrimina-

tory auction with capacity limits. Capacity limits imply that sometimes the second-highest or third-highest bid will win. But the discriminatory part of the mechanism ensures that a winner always pays his bid. We do not have to consider bid strategies based on second-price or third-price formats where bids will typically be closer to valuations than under the first-price format. In a third-price format, equilibrium bids can actually be higher than valuations.

Modelling the bid strategies with random multi-unit demand is messy. In addition to requiring a distribution over valuations, one needs a distribution over demand in order to model the appropriate number of bidders. Making the strategies excessively complex also leads us to question how appropriate the models are in reflecting real-world behavior. We proceed under the assumption that the bid function at single-object auctions captures the essential strategic considerations, but sometimes will underestimate bid shaving.<sup>5</sup> Bids will still be below valuations since it is a first-price auction, bids will still be increasing in  $\mathcal{N}$  and  $r$ , and bids will still depend on the prior beliefs of other bidders valuations represented by  $F_V$ . The main difference is that expected revenue may depend on a lower order statistic from  $F_V$  than in the single-object case where expected revenue is given by  $\mathcal{E} [V_{(\mathcal{N}-1:\mathcal{N})}]$ .

**Effect on optimal reserve price.** Finally, what effect does increased bid shaving have on the formula for the optimal reserve price given by equation (5)? Valuations will not change under multi-object sales since we assume constant marginal values, only strategies may change. The optimal reserve price depends on the distribution of valuations, not the distribution of bids. Consequently, the optimal reserve price will not change when the equilibrium solution predicts increased bid shaving. In fact, the importance of setting a reserve price is likely to increase when bid shaving increases.

The optimal reserve price depends on the shape of  $F_V$  and  $f_V$ . One might wonder if the finer details of the distributions have an unpredictable impact on calculating the optimal reserve price  $r^*$  based on an estimate  $\hat{F}_V$  that underestimates the true  $F_V$ . Basic economic and statistical reasoning suggests the answer is no. If buyers have constant marginal values for objects at a simultaneous-independent discriminatory auction, then the optimal reserve

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<sup>5</sup>We say *sometimes*, because in several auctions there is actually just one lot for sale and, at other times, some or several buyers demand the total supply.

price  $r^*$  based on the true  $F_V$  will not be less than the estimated optimal reserve price  $\hat{r}^*$  based on  $\hat{F}_V$ . The reasoning supporting this proposition is as follows: First, demand is fixed. Consider one unit offered at a single-unit, first-price auction. The expected revenue is  $\mathcal{E}[V_{(\mathcal{N}-1:\mathcal{N})}]$ . Next, increase demand by introducing several units. It follows from fundamental economic theory and the principle of purposeful behavior that with demand fixed and supply increased, the new equilibrium price cannot be higher than under the single-unit format.

What are the consequences if we model bids by (2) and derive the optimal reserve price by (5), and the actual bid shaving taking place is more severe than our model predicts? In that case, identifying values by incorrectly assuming the single object format underestimates values, since actual bid shaving is more severe than what follows from the theoretical bid shaving factor we use. In consequence, our estimate of  $\hat{F}_V$  is first-order stochastically dominated by the true  $F_V$ —i.e., for all  $v \in [v, \bar{v}]$ ,  $\hat{F}_V(v) \geq F_V(v)$ . Thus, the estimated inverse hazard rate  $(1 - \hat{F}_V)/\hat{f}_V$  first-order stochastically dominates the true  $(1 - F_V)/f_V$ . Since the optimal reserve price at the single-object auction is defined by

$$r - v_0 - \frac{[1 - F_V(r)]}{f_V(r)} = 0,$$

it follows that  $r^* \geq \hat{r}^*$ . This is formally shown in appendix A.2. Thus, the estimated reserve price will represent a lower bound for the optimal reserve price as far as model misspecification is concerned. A statistical estimation error will, however, always be present.

### 3.3 Symmetry and independence of valuations

A key element of auction models is to what extent bidders' valuations of the auctioned good are correlated. Two polar cases are private-values and common-value. Valuations are private if they are not dependent; i.e., the value one bidder places on a good does not depend on how other bidders value it. Information on competitors' valuations do not influence one's own valuation of the good. On the other hand, valuations are common if all value the good at the same price. A distinguishing characteristic between the two concepts concerns the degree of certainty of valuations. Typically, a common value is uncertain. The prime example of this situation is oil

companies bidding for the right to exploit oil tracts with uncertain contents. Most real-world auctions have a private-values and a common-value element. In the following, however, we shall argue that the auction under study is predominantly a private-values auction. A closer look at what characterizes buyers' input and output markets and a study of what kind of uncertainty that is present, is necessary to proceed.

To begin, consider the output market. The buyers are food producers who use the fish as an input to various end-products. If bidders face an uncertain future end-product price at the time of bidding for raw material, then a common-value element is present. The price uncertainty in the end-product markets is, however, probably relatively small. For most of these products, established competitive prices are only to a small degree sensitive to fluctuations in the supply side of inputs in one market. The end-products face competition both from other complementary food products, and from similar products from food producers who get their inputs in other markets. In case the end-product is a conserved good, like canned products, the price variability is known to be small. In case the fish are shipped unprocessed as fresh or frozen to export markets, the time span between raw costs and revenues is, in this industry, small. Thus, presumably, the true value of the end-product is quite certain, and the common-value element is negligible.

There *is* a common-value element in the cost structure in the sense that producers all face some common costs and constraints in production. Wage levels and capital costs may not differ that much, and public taxes and charges are similar for all. However, there is no particular uncertainty with respect to these costs. The interesting part of the common-value aspect, the uncertainty involved, is, hence, not an issue. The common-value part of costs defines a base level for all producers.

If we are willing to invoke a certain degree of rationality on the sellers, an additional argument for private valuations follows from the analysis of Levin and Smith [11]. As mentioned, they showed that if valuations are correlated, the optimal reserve price converges, often rapidly, to the seller's private value when the number of bidders increases. The fact that sellers commit to a reserve price that is considerably higher than their private value, suggests that buyers' valuations are predominantly private.

Having argued that valuations are private, we next turn to the question of whether bidders are symmetric in the sense that bidders' valuations are

identically distributed and drawn from the same distribution. This does not mean that bidders are identical, just that their valuations are drawn from the same distribution. The distribution is said to encompass all relevant information bidders have about their competitors. Bidders entertaining a specific bid make assumptions about the probability that competitors' valuations exceed a certain level. The assumption of identically-distributed valuations entails that a bidder has no reason to believe that a particular competitor has a certain valuation with higher probability than others. The assumption has important consequences for the empirical analysis. The theoretical model we have employed is a one-shot game. In order to analyse auction games empirically we have to aggregate observations from a sequence of auctions. But how can we defend the assumption that valuations are identically distributed over time?

To begin, we notice that such an assumption is, at most real-world markets with repeated auctions, a simplification. Over time, given the information previous auctions reveal with respect to individual bidder behavior, it is likely that some learning takes place. During this learning process, bidders' information sets might be transformed as follows: At first, they start with a crude assumption on the distribution of valuations, associated with a common distribution with relatively large or wide scale and location parameters. Later, as experience is growing, the information set changes to a more sophisticated representation where individual bidders or groups of bidders are associated with more concentrated individual distributions of valuations.

If bidders' cost structures differ in a systematic way, then there will be asymmetries. We invoke the paradigm of competitive markets, and assume that buyers, operating within their capacity, will more or less have the same expected long-run profit margins. But if variation in buyers' profit margins is relatively small, then why do they submit different bids? To answer, we must pay attention to the one important specific characteristic of production: the short-term capacity constraints of buyers. Since the object for sale is dead fish, processing and freezing must take place within a short time span in order to avoid spoilage. A buyer who, at the time when a specific auction is held, has lots of raw material, will not be as keen to bid high as a buyer who suffers shortage of raw material. Private values will differ in this respect both among different buyers and for a given buyer over time. The dynamics

of the market will shift a buyer's private value within the distribution from auction to auction. Since variations in short-term capacities are random, using a common distribution of valuations, seems especially appropriate in this market.

### 3.4 Risk attitude

Next, we turn to the question of whether bidders are risk averse or risk neutral. Although most applied studies simply assume risk neutrality, we devote some space to discuss the assumption. Risk aversion is the normal case; several studies, especially in the finance literature, have shown that human decisions are best modelled by use of risk aversion. Various notions of risk aversion have been developed in the literature. Constant risk aversion implies that bidders have the same attitude or aversion for risk irrespective of the amount at stake, while increasing risk aversion means that the negative preference for risk increases with the amount.<sup>6</sup> Recall from auction theory that a bid may be seen as composed of two factors. A bidder will balance (1) the probability of winning or equivalently the risk of losing with, (2) the realized profit if winning. These factors work in opposite ways; an high bid increases the probability of winning while at the same time reducing the profit margin. Riley and Samuelson [16] showed that a bidder with increasing risk aversion will place more weight on the probability of losing-factor when the amount at stake increases compared to a risk neutral bidder. Consequently, since the probability of losing increases with decreasing bids, equilibrium bids are higher if we model bidders as risk averse rather than risk neutral.

To begin, we assume that bidders at this auction are risk averse. In the benchmark model, on the other hand, bidders are modelled as risk neutral. This is a simplifying assumption making the model tractable, since introducing risk aversion complicates the model substantially. A possible defense for the simplifying assumption of risk neutrality is that, if at a given auction, the risk involved is marginal, then risk neutrality may be an acceptable modelling strategy.

The central question then becomes: How risk averse are bidders at the mackerel auction? We shall argue that there is relatively little risk involved

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<sup>6</sup>A distinction is made between *absolute* and *relative* risk aversion. The above explanation refers to the concept of absolute risk aversion.

at the mackerel auction. Bidding on a single lot does not entail a large financial burden for the winner, relative to the total turnover.<sup>7</sup> The argument rests on the fact that increasing risk aversion seems to be the norm. People are generally not risk averse with respect to small amounts, but less inclined to gambles when large amounts, relative to wealth, are involved. This is supported by the close to universal fact that people do not insure small value items, while most people purchase insurance for valuable assets.

Moreover, during the season, many catches are offered on the market within frequent time intervals; in the peak season, up to four auctions are held each day. In addition, plants have the opportunity to buy other species. The risk of losing at a given auction is therefore offset by other opportunities. We conclude that risk neutrality seems to be a reasonable approximation to risk aversion of small order.

### 3.5 Fixed and known number of participants

An important assumption in most auction models is that the number of potential bidders,  $\mathcal{N}$ , is known. Moreover, when analysing auction data empirically using the structural approach, we need to aggregate comparable auctions. This raises two questions to be addressed by the researcher. First, is the assumption of a known  $\mathcal{N}$  plausible at a single auction? Second, is  $\mathcal{N}$  stable over sequential auctions?

At single-shot auctions, assuming a certain number of participants may in some instances be a bit off the wall. One case in question is where a large and complicated sale involves substantial pre-contract costs. Firms considering bidding will have to weigh the potential benefits and the costs from participating. Under this scenario, bidder participation can be explained by stating that, in equilibrium, the expected profit from participating is equal to the sunk costs that pre-contract efforts involve; see French and McCormick [3]. In our case, we have frequently repeated auctions, where the competition for raw materials is routinely undertaken. Participation costs are likely to be negligible.

Another case in question, is when there is a general uncertainty about the number of competitors. This can be modelled by introducing a distribution over  $\mathcal{N}$ . Harstad, Kagel, and Levin [7] showed that at first-price auctions,

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<sup>7</sup>To be more precise, we should compare the risk involved with the plant owners' wealth since this is the relevant factor in the theory of risk; see, for example, Gollier [4].

with risk neutral agents, the unique symmetric equilibrium-bid function is a weighted average over the bid functions with a known  $\mathcal{N}$ . Formally, if  $b_{\mathcal{N}}$  is the bid function with a certain number of bidders  $\mathcal{N}$ , then the bid function with uncertainty in the number of participants at the auction is:

$$b(v) = \sum_{\mathcal{N}} w_{\mathcal{N}}(v) b_{\mathcal{N}}(v), \quad (9)$$

where the weight  $w_{\mathcal{N}}$  is the probability of  $\mathcal{N}$  bidders conditional upon winning with bid  $b_{\mathcal{N}}$ . Each bid  $b_{\mathcal{N}}$  is the standard Bayes–Nash equilibrium bid presented in equation (2). Obviously, the weighted bid over  $\mathcal{N}$  will be lower than a bid based on the maximum  $\mathcal{N}$ .

In the present market, we chose to analyse one delivery sector, one of the most frequently observed. Lots offered in this sector will have a stable set of potential bidders since it is an industry with high entry costs; the competitors know about each other. It is a fact, however, that the number of actual bidders  $N$  varies from one auction to another. Formally, this could be defended by saying that bidders' valuations vary over time, within the same distribution  $F_V(v)$ . A specific draw from the distribution of valuations results in  $N$  bidders with valuations above the reserve price. It seems, however, somewhat unrealistic to assume that bidders' strategies are not affected by the observed variability in  $N$ .

Another approach is to assume that the number of potential bidders varies from auction to auction because some plants are not in a buyer position due to capacity constraints; i.e., their valuations drop to zero in this case. Although bidders do not have exact information on competitors' capacities at a given time, the total supply will be an indicator of whether many capacity constraints are binding. This situation seems to invite bidders to form their bids on a distribution over  $\mathcal{N}$  like the model considered by Harstad, Kagel, and Levin [7]. We analyse the market under the condition that the number of participants is the full set of observed potential bidders; i.e., we use the maximum  $N$  observed in the empirical specification of the model. If bidders use a weighted average bid to account for numbers uncertainty, then their bids will be lower than what is predicted by our model. The error is one-sided, and the consequence is that valuations will be consistently underestimated. We see that the error is of the same nature as discussed in section 3.2. Thus, we rely on the same stochastic

order result (see page 14) that says that the estimated optimal reserve price will represent a lower bound on the true optimal reserve price.

## 4 Empirical specification

We proceed by explaining how we can utilize the identification result of equation (8) to estimate bidders' private values, and the distribution of valuations, empirically. The approach is based on the idea that these estimates represent a lower bound on true values.

The strategy for finding  $F_V(v)$  and  $f_V(v)$ , necessary to compute the optimal reserve price  $r^*$ , is to follow the two-step estimator of Guerre et al. [5]: First, in order to uncover unobservable private values using relation (8), we need estimates of  $g_B(b)$ ,  $G_B(b)$ ,  $\mathcal{N}$  and  $F_V(r)$ . Straightforward estimators of  $\mathcal{N}$  and  $F_V(r)$  can be obtained, while the estimation of  $G_B(b)$  and  $g_B(b)$  are complicated by the presence of a binding reserve price, since observed bids represent a truncated sample of the full set of potential bids without a reserve price. A transformation of bids is necessary in order to proceed. Once we have obtained the estimates, we calculate pseudo-valuations by using a modified version of relation (8) on page 7, see relation (13) below. The next step is then to use the calculated pseudo-valuations to estimate the conditional functions  $F_{V|V \geq r}(v|v \geq r)$  and  $f_{V|V \geq r}(v|v \geq r)$ . Finally, we transform the conditional probability functions to estimated unconditional functions  $F_V(v)$  and  $f_V(v)$ .

### 4.1 Estimation of the bid distribution

A technical problem concerning the estimation of  $g_B(b)$  and  $G_B(b)$  is that the density  $g_B(b)$  is unbounded at  $b = r$  since  $\beta(v < r) = 0$ . When the data at hand is not unbounded—in our case observed bids are bounded—then straightforward kernel density estimation will fail, since the kernel estimate near the boundary is not consistent; see Simonoff [19, section 3.2.1]. In our setting, this means that  $g_B(b) \rightarrow \infty$  as  $b \searrow r$ . Guerre, Perrigne and Vuong [5] note that  $g_B(b)$  is proportional to  $1/\sqrt{b-r}$  when  $b \searrow r$ . The solution they suggest is to transform observed bids  $B$  to a variable

$$B(r) = \sqrt{B-r}. \tag{10}$$

Under this transformation, the cdf and pdf of the transformed bids,  $G_{B(r)}$  and  $g_{B(r)}$ , can be expressed in terms of the corresponding functions of observable bids,  $G_B$  and  $g_B$ , as:<sup>8</sup>

$$G_{B(r)} [b(r)] = G_B [r + b(r)^2] \quad (11)$$

and

$$g_{B(r)} [b(r)] = \frac{dG_B}{db(r)} = 2b(r) g_B [r + b(r)^2]. \quad (12)$$

From (10) we have that  $b = b(r) + r^2$ . Substituting this, together with the expressions in (11)–(12), into (8) yields the result that valuations are identified by

$$v = b(r)^2 + r + \frac{2b(r)}{(\mathcal{N} - 1)} \left\{ \frac{G_{B(r)} [b(r)]}{g_{B(r)} [b(r)]} + \frac{F_V(r)}{g_{B(r)} [b(r)] [1 - F_V(r)]} \right\}. \quad (13)$$

Let  $T$  be the number of sampled auctions, and  $N_t$  the number of observed bids at auction  $t$ . A straightforward estimator of  $G_{B(r)}$  is the empirical cdf of the transformed observed bids

$$\hat{G}_{B(r)} [b(r)] = \frac{1}{T} \sum_{t=1}^T \frac{1}{N_t} \sum_{i=1}^{N_t} \mathbf{1} [B_{it}(r) \leq b(r)].$$

We estimate the truncated density function of bids by a kernel-smoothed density estimator

$$\hat{g}_{B(r)} [b(r)] = \frac{1}{Th_g} \sum_{t=1}^T \frac{1}{N_t} \sum_{i=1}^{N_t} \kappa \left[ \frac{B_{it}(r) - b(r)}{h_g} \right] \quad (14)$$

where  $\kappa(\cdot)$  is a kernel function satisfying some standard assumptions, and  $h_g$  is a smoothing parameter, also called the bandwidth or window width. We discuss the appropriate choice of kernel function and bandwidth in section 5.2.

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<sup>8</sup>  $G_{B(r)} [b(r)] = \Pr [\sqrt{B - r} \leq b(r)] = \Pr [B \leq b(r)^2 + r] = G_B [r + b(r)^2]$ .

## 4.2 Estimation of $\mathcal{N}$ and $F_V(r)$

With no reserve price, the number of potential bidders  $\mathcal{N}$  equals the number of observed bidders. When introducing a reserve price,  $\mathcal{N}$  is unobserved since bidders with valuations below  $r$ , do not bid. We observe the number of actual bidders  $N$ , a subset of the number of potential bidders. Following Paarsch and Hong [15], we describe the relationship between  $\mathcal{N}$  and  $N$  and present suitable estimators for  $\mathcal{N}$  and  $F_V(r)$ . They noted that the number of active bidders is the sum of a Bernoulli sequence:

$$N = \sum_{i=1}^{\mathcal{N}} I_i \quad \text{where} \quad I_i = \begin{cases} 1 & \text{if } v_i \geq r, \text{ with probability } [1 - F_V(r)]; \\ 0 & \text{if } v_i < r, \text{ with probability } F_V(r). \end{cases}$$

Thus,  $N$  has a binomial distribution with parameters  $\mathcal{N}$  and  $[1 - F_V(r)]$ , and the probability mass function is then:

$$f_N(n) = \binom{\mathcal{N}}{n} F_V(r)^{\mathcal{N}-n} [1 - F_V(r)]^n, \quad n = 0, 1, \dots, \mathcal{N}.$$

A natural estimator for  $\mathcal{N}$  is

$$\hat{\mathcal{N}} = \max_{t=1, \dots, T} N_t. \quad (15)$$

It can be shown that  $\hat{\mathcal{N}}$  converges almost surely to  $\mathcal{N}$ .

In order to find an estimator for  $F_V(r)$ , we note that  $[1 - F_V(r)]$  is one of the parameters in the probability mass function of  $N$ . Using the fact that the expectation of a binomially distributed variable is equal to the product of its parameters, we get an expression for  $F_V(r)$ :  $\mathcal{E}(N) = \mathcal{N} [1 - F_V(r)]$ , or  $F_V(r) = 1 - [\mathcal{E}(N) / \mathcal{N}]$ . We estimate  $\mathcal{E}(N)$  by the sample mean  $\bar{N}$  and  $\hat{\mathcal{N}}$  by expression (15). Hence, an estimator for  $F_V(r)$  is

$$\hat{F}_V(r) = 1 - \frac{\bar{N}}{\hat{\mathcal{N}}} = 1 - \frac{T^{-1} \sum_{t=1}^T N_t}{\left( \max_t N_t \right)}. \quad (16)$$

## 4.3 Uncovering valuations and estimating $f_V$ and $F_V$

Having presented estimators for  $G_{B(r)}$ ,  $G_{B(r)}$ ,  $N$ , and  $F_V(r)$ , we are now in a position to calculate an estimate of the valuations that bidders base their bids on, the so-called pseudo-values. For each observation  $it$ , we use (13)

and recover valuations from bids by

$$\begin{aligned} \hat{V}_{it} &= B_{it}(r)^2 + r \\ &+ \frac{2B_{it}(r)}{(\hat{N} - 1)} \left\{ \frac{\hat{G}_{B(r)}[B_{it}(r)] [1 - \hat{F}_V(r)] + \hat{F}_V(r)}{\hat{g}_{B(r)}[B_{it}(r)] [1 - \hat{F}_V(r)]} \right\}. \end{aligned} \quad (17)$$

We now want to find the density and distribution of estimated valuations in order to reach the stated goal of using equation 8 to estimate a lower bound on the optimal reserve price. The truncated pdf of valuations is again estimated by a kernel estimate

$$\hat{f}_{V|V \geq r}(v|V \geq r) = \frac{1}{Th_g} \sum_{t=1}^T \frac{1}{N_t} \sum_{i=1}^{N_t} \kappa \left( \frac{\hat{V}_{it} - v}{h_g} \right), \quad (18)$$

and an estimate of the truncated cdf is given by the empirical cumulative distribution function of estimated valuations

$$\hat{F}_{V|V \geq r}(v|V \geq r) = \frac{1}{T} \sum_{t=1}^T \frac{1}{N_t} \sum_{i=1}^{N_t} \mathbf{1}(\hat{V}_{it} \leq v).$$

Finally, to arrive at the unconditional probability functions, we use the following transformation:

$$\hat{f}_V(v) = \hat{f}_{V|V \geq r}(v|V \geq r) [1 - \hat{F}_V(r)]$$

and

$$\hat{F}_V(v) = \hat{F}_{V|V \geq r}(v|V \geq r) [1 - \hat{F}_V(r)]. \quad (19)$$

Notice that this estimation procedure only gives us information about the shape of  $f_V$  and  $F_V$  above the reservation price. Below  $r$ , we have no observed bids, and, thus, no information to infer  $f_V$  and  $F_V$  from. The practical consequence is that, if the reserve price in use is initially set above the optimal level, then we cannot estimate the correct level. The only conclusion we can reach is that the reserve price should be somewhere between  $v_0$  and  $r$ .

## 5 Estimation and results

### 5.1 The data sample

The construction of the dataset is documented in the appendix of the thesis. For the 2003–4 season, beginning in mid-August and ending in February, we have observations of all submitted bids. In order to have a well-defined market with a stable set of potential buyers, we chose to analyse one of the delivery sectors most frequently observed. The sector has the city Bergen with numerical code 19 as the southern border and a specific location in the county of Møre represented by numerical code 25 as the northern border. We then chose rather homogenous catches with a stated reserve price equal to NOK 5.25. A reserve price of 5.25 refers to fish with average weight equal to or above 500 grams. This is the weight class of the largest fish, and the most important product, both in terms of the number of offered catches and in terms of total quantity. Consequently, since the reserve price and the realized price are increasing with average weight, it is also the most important weight class with respect to revenues.

In principle, buyers outside the delivery sector are entitled to submit bids. If an outside bidder has the highest bid, the seller is free to refuse it. A few bidders seem to bid routinely on outside catches. Normally, the extra cost incurred in serving an outside bidder with the highest bid outweighs the increased revenue. In fact, of all the catches offered in 2003–4 (all sectors and all weight classes), only 2.4 percent of the catches were allocated to an outside bidder, and in our sample of catches an outside bidder is never allocated a catch. We remove outside bids from our sample since they are not considered realistic bids, and since including them, would break down the assumption of a stable set of potential bidders. This gives us a sample of 57 auctions, 97 objects for sale and a total of 708 submitted bids. The distribution of the number of catches per auction, which has a reserve price equal to 5.25, in the relevant sector is:

Table 1: Distribution of lots per auction

Number of lots per auction:	1	2	3	4	5	7
Frequency:	35	13	2	2	3	1

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We see that 61 percent of the auctions have only a single catch offered in this delivery sector at the reserve price. However, there will, in some cases, be catches with the same reserve price offered to other sectors. Some of these will partly overlap the sector under study here. In consequence, some of the buyers in sector 19–25 will have opportunities to bid on other catches. In addition, catches with fish of lighter weight and a lower reserve price will also be offered both to the same sector and to other partly overlapping sectors. One particular auction which contained 8 relevant objects and that otherwise met the two requirements—catches offered to (1) sector 19–25 with (2) a reserve price equal to 5.25—were not included in the data sample since, at this auction, several similar catches were offered to a partly overlapping sector. If we concentrate on the current sector, at the sampled auctions, 25 objects are not included because they do not meet the weight requirement and have a lower reserve price. Our sampled objects comprise 80.0 percent of the total objects and 79.6 percent of the quantity offered to this sector at the same auctions. In terms of realized revenues, the equivalent measure is 81.3 percent.

Laffont, Ossard and Vuong [10], in their analysis of a French eggplant auction market, made a point of sampling auctions where only one lot was offered each day to avoid the influence of the “dynamics of the market” on bidder strategies. It is not obvious that such a sampling strategy is sufficient to avoid the bulk of the universal noise that every empirical analysis is ridden with when confronting real-world data with theoretical models. Every market is influenced by substitutes and complements and several general variables. Some way or another, we have to define our market clearly. We chose to emphasize that the sample consists of homogenous products with identical reserve prices and a stable set of potential buyers; these are the requirements of the estimation procedure used. The influence of other, dissimilar catches offered in the market is considered of less importance. The competition in the market, characterized by the excess demand and the number of competitors, narrows down the strategy space of bidders, both in the given market and in the substitute goods markets. Thus, the errors invoked in estimated valuations are relatively small compared to environments where competition is weaker. Moreover, the error is one-sided, and we can interpret our estimated valuations as lower bounds on true valuations.

Some summary statistics of bids are reported in table 2. The first column

Table 2: Summary statistics of bids

	Max bids	Winning bids	All bids
Sample size	97.00	94.00	708.00
Minimum	5.25	5.25	5.25
25th percentile	6.65	6.58	6.23
50th percentile	7.15	7.10	6.88
75th percentile	7.42	7.39	7.16
Maximum	7.94	7.94	7.94
Mean	6.97	6.93	6.68
Standard deviation	0.60	0.57	0.63
Skewness	-1.25	-0.94	-0.73
Kurtosis	3.98	3.18	2.52

in table 2 shows the statistics for the maximum bid of an object, and the third column shows the same statistics for all bids. The second column shows the statistics of the bid that was allocated the object; recall that some objects will go to lower order bids if the high bidder has reached his stated limit in terms of quantity. The number of observations for this variable is 94, since there are three objects in the dataset that went unsold although they received bids. While the mean of the maximum bid is 4.19 percent higher than the mean of all bids, it is only 0.58 percent higher than the mean of winning bids.

## 5.2 Choice of kernel and bandwidth

In equations (14) and (18), we use a kernel function  $\kappa(\cdot)$  to obtain the conditional probability functions  $\hat{g}_{B(r)}$  and  $\hat{f}_{V|V \geq r}$ . A kernel function is defined to be symmetric around 0 and must integrate to one. Since the kernel is a density function, the kernel estimate will also be a density. For further characterizations of some standard requirements for kernel functions, we refer to Härdle [6]. Several kernel functions are known to produce reliable results, the exact choice is not critical since the differences in efficiencies are very small for the commonly used kernel functions. We used the Epanechnikov

kernel defined by  $\kappa(u) = \frac{3}{4}(1 - u^2)\mathbf{1}(|u| \leq 1)$ .

The choice of bandwidth, on the other hand, is critical for the kernel estimator. A bandwidth that is too narrow, will oversmooth the density function, while a broad bandwidth will undersmooth it. No universal agreement on the optimal choice of bandwidth seems to exist. Silverman [18] reports a rule-of thumb of  $h_g = 1.06\hat{\sigma}S^{-1/5}$ , where  $\hat{\sigma}$  is the standard error of the sample of size  $S$ . This is frequently used; for example, Guerre et al. used it in their simulation analysis demonstrating their two-step estimator. We used this bandwidth as well, although a data driven bandwidth selector, such as the one proposed by Sheather and Jones [17] is preferable.

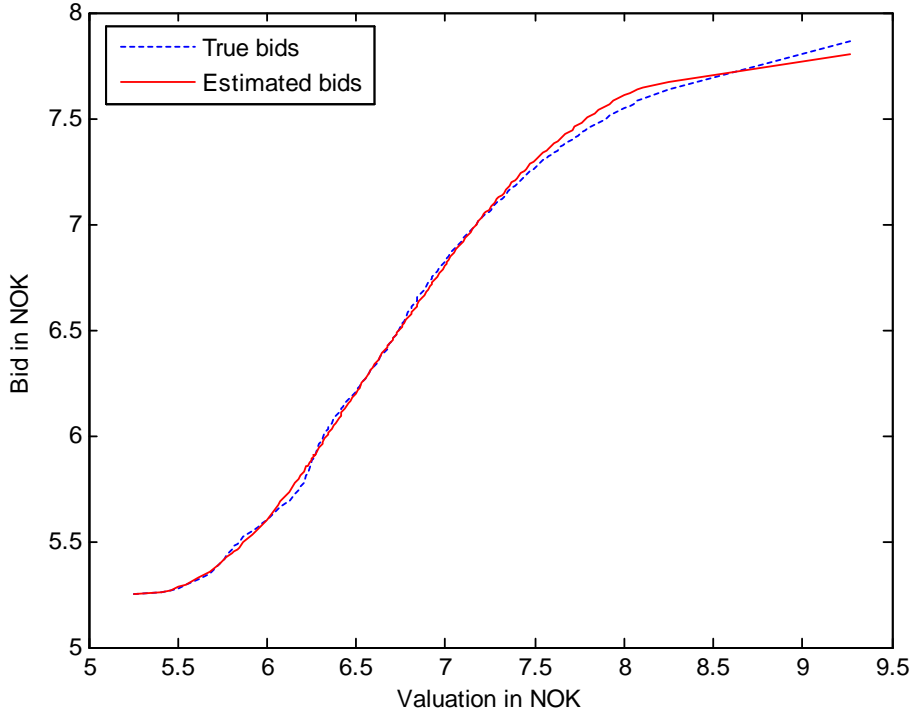
### 5.3 Results

**Fit between true bids and estimated bids.** A key output of the estimation procedure are the estimated underlying valuations from equation (17) and the associated cumulative distribution function from equation (19). How well do our estimates of valuations explain bids given our model? Using our estimated valuations, we can estimate bids by using the theoretical bid function of equation (2) and compare them to the observed bids. In figure 1, we plot the estimated valuations against both the true observed bids and the estimated bids. The fit between true and estimated bids is quite good. The estimation error is below 1 percent for all observations; the range is from  $-0.52$  percent to  $0.98$  percent.

**The optimal reserve price.** The number of average bidders per catch is around 7 (arithmetic mean = 7.30). The maximum number of bidders observed at an auction, is 14. According to (16) this gives us  $\hat{F}_V(r) = 0.48$ . When we estimate the cumulative distribution function and population density function of valuations, we obtain the functions  $\hat{f}_V(v)$  and  $\hat{F}_V(v)$  shown in figure 2.

With estimates of  $F_V$  and  $f_V$ , we can numerically solve for the optimal reserve price. Since  $r^*$  depends on the seller's reservation value  $v_0$ , we must account for this in the calculation. If a catch goes unsold, it might be sold for meal production instead of for human consumption. In that case, it obtains a price considerably lower than the current reserve price. In figure 2, we have indicated a lower bound for  $r^* = 5.98$  when  $v_0 = 1$ . This suggests that the reserve price should be raised by at least 13.9 percent.

Figure 1: True and estimated bids

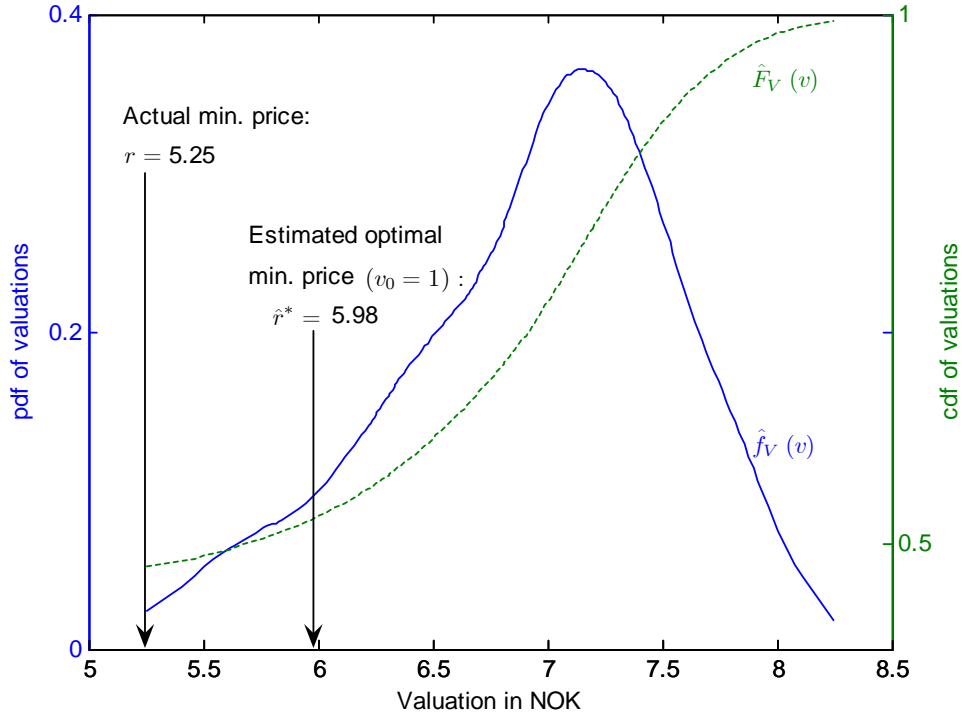


We choose to calculate  $\hat{r}^*$  for different assumptions of  $v_0$ . Obviously,  $\hat{r}^*$  is increasing in  $v_0$ . The relationship between  $\hat{r}^*$  and  $v_0$  is shown in figure 3.

**Effects on shaving factor and revenues.** Next, we turn to the question of how an increase in the reserve price will affect revenues, if implemented. Let us first look at the degree to which shaving factors are reduced when an higher reserve price is introduced. With as many as 14 potential bidders, we expect this market to be quite competitive, so the shaving factors will likely be moderate. Using the estimated valuations and true bids for all 708 observations, we find that the average shaving factor of bids is as low as 3.85 percent (minimum 0 percent and maximum 27.16 percent).

In order to compare actual and counter-factual shaving factors, we focus on the lots that are sold in the latter case, and the case where  $v_0 = 1$  and  $\hat{r}^* = 5.98$ . In table 3, we report the estimated shaving factors for all bids

Figure 2: Estimated probability functions



and for only the winning bids. Actual shaving factors with a reserve price equal to 5.25 are, for all relevant statistics, larger than the shaving factors observed when the reserve price is set to 5.98.<sup>9</sup> This is to be expected; the point of raising the reserve price is to decrease shaving factors. The difference in the mean shaving factor is 0.64 percent for all bids and 0.56 percent for winning bids. This indicates that the possible gains from raising the reserve price are moderate.

Before we conclude that the seller should raise the reserve price, we examine possible efficiency effects of raising the reserve price by calculating counter-factual bids and revenues. We have seen that an increased reserve price decreases the shaving factor. When calculating counter-factual revenue, we respect the realized allocation. For example, if a catch was sold to

<sup>9</sup>The only exception is for the maximum statistic case. The estimate of the maximum valuation bid seems to underestimate the true bid.

Figure 3: The optimal reserve price as a function of seller's own valuation,  $v_0$

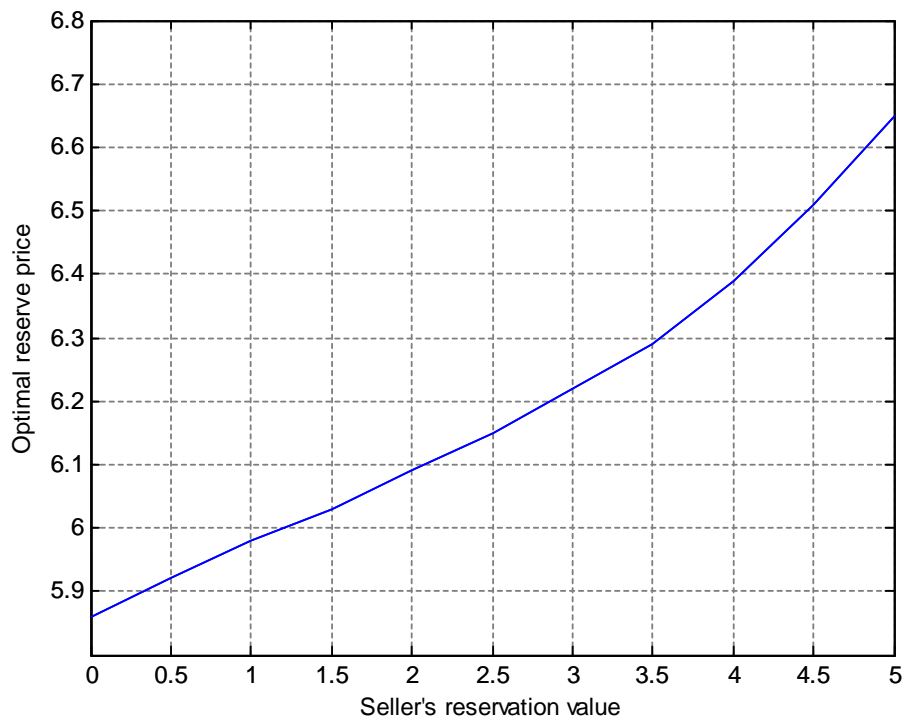


Table 3: Shaving factors in percent,  $v_0 = 1$ 

	All bids <sup>a</sup>		Winning bids <sup>b</sup>	
	Actual $r = 5.25$	Estimated $r = 5.98$	Actual $r = 5.25$	Estimated $r = 5.98$
Minimum	2.42	0.46	2.43	1.59
25th percentile	2.56	2.50	2.79	2.59
50th percentile	3.14	2.91	3.63	3.16
75th percentile	4.52	3.49	4.65	3.60
Maximum	27.16	28.89	27.16	28.89
Mean	3.72	3.08	4.14	3.58
Standard deviation	1.71	1.38	2.75	2.84

<sup>a</sup> Number of observations: 641

<sup>b</sup> Number of observations: 91

the second-highest bid, then the counter-factual bid from that same bidder is used. The estimation procedure of underlying valuations and corresponding counter-factual bids ensures that the ranking between counter-factual bids is not altered relative to the true bids. In cases where valuations drop below the new reserve price, however, all the associated bids will equal zero. The optimal reserve price is derived by balancing the risk of having unsold objects with the gains from reduced shaving on the objects that do attract bids. At one-shot auctions, the outcome can indeed be suboptimal in the sense that an object may go unsold. However, at repeated auctions with a sufficiently large sample, it is likely that the realized outcome is close to the expected revenue predicted.

Counter-factual revenue depends on what optimal reserve price  $\hat{r}^*$  we use, which, in turn depends on the seller's true reservation value  $v_0$ . Previously, we argued that a reservation value of one is likely. In table 4, the estimated percentage change in revenues is reported for a range of different reservation values.

The important result in table 4 is column 3 where the percentage change in revenue from going from a reserve price equal to 5.25 to the reserve price in column 2, is reported. The result is that total revenues slightly decrease

Table 4: Estimated revenue effects

$v_0^a$	$\hat{r}^{*b}$	Change in revenue (%)		
		All lots	Sold lots	Unsold <sup>c</sup>
0.00	5.86	-0.33	0.44	3
0.50	5.92	-0.23	0.47	3
1.00	5.98	-0.12	0.50	3
1.50	6.03	-0.03	0.52	3
2.00	6.09	-0.18	0.55	4
2.50	6.15	-0.05	0.58	4
3.00	6.22	0.14	0.66	4
3.50	6.29	-0.11	0.69	6
4.00	6.39	-1.56	0.63	10
4.50	6.51	-1.54	0.66	14
5.00	6.65	-2.04	0.58	19

<sup>a</sup>  $v_0$ : Seller's own valuation.

<sup>b</sup>  $\hat{r}^*$ : Estimated optimal reserve price.

<sup>c</sup> Number of unsold lots due to increase in the reserve price.

for most  $v_0$  below 4. This is due to inefficiencies that a raised reserve price entails. In the sample, we have three unsold lots. In column 5 of table 4, the additional unsold lots, due to an increase in a reserve price, are reported. We see that some additional 3–4 lots go unsold in the counter-factual case when  $v_0$  is below 3.50. For reservation values above 3.50, the effect of inefficiencies is more severe, both in terms of unsold lots, and in terms of reduced revenue. In this sample, and given our assumptions, the total effect of raising the reserve price is that the costs associated with unsold lots that will have to end up as meal production at value  $v_0$ , outweigh the increase in revenues from higher bids on the sold lots. The *ex post* optimality of the calculated reserve price rests on large sample properties. Thus, care must be taken when considering changes in the reserve price in a given market with a limited number of objects for sale.

We may risk overstating the importance of inefficiencies due to unsold lots in this particular market. Mackerel in September and October (the peak season) are quite robust and can retain their high quality for several auction rounds. Thus, very few catches end up in meal production; unsold catches in one auction tend to being sold in a later auction.

The differences between actual and counter-factual revenues are, however, very small. It is fair to say that our estimates of valuations are conservative. As discussed previously, both the general demand reduction effect due to the multi-sales format, and the uncertainty with respect to the number of bidders—see equation (9)—make the estimate a lower bound of true valuations. If true valuations were sufficiently underestimated, then possible inefficiency effects would not bind. In column 4 of table 4, the revenue effects on those lots that will be sold under both regimes, are reported. The increase in total revenue is somewhere between 0.44 percent and 0.69 percent. Admittedly, this is not a substantial increase, but even a small increase in revenues is worth considering since the absolute amounts involved in a market with a turnover of several billion NOK, are large.

## 6 Concluding remarks

Theory suggests that there may be a potential for raising revenues by adjusting the reserve price upwards. In the long run, we estimate an increase in revenues of about 0.50 percent. Even though the observed reserve price is

quite far from the estimated optimal, the revenue effects are moderate, and in that sense, the reserve price in use is, probably, not far from the target.

The analysis rests on the assumption that the theoretical model captures the driving elements of bid shaving behavior at these auctions reasonably well. Some assumptions of the analysis may oversimplify certain elements. First, the two-step estimator of Guerre et al., on which our analysis is based, considers bidders' prior beliefs to be symmetric. An interesting extension of the model is provided by Brendstrup and Paarsch [1], which allows for asymmetric bidders. The data requirements for computing individual probability functions for each bidder, may, however, prove to be too demanding in our case. Second, the analysis may benefit from a more sophisticated treatment of the number of potential bidders. The number of active bidders varies considerably across auctions. Given the high number of potential bidders we use, the market may appear more competitive than it actually is for some lots. Finally, although we have tried to sample homogenous lots, heterogeneity may be present. Investigating heterogeneity requires a parametric rather than a nonparametric, structural approach.

## A Appendix

### A.1 Probability distributions of bids

In this section, we show how the formula for the probability distributions of bids reported in equations (6) and (7) are derived. The cumulative distribution function (cdf) of observed bids, when a binding reserve price is present, is conditional on private values being above  $r$ :

$$G_B(b) = \Pr[\beta(v) \leq b | v \geq r] = \Pr[v \leq \beta^{-1}(b) | v \geq r] = F_V[\beta^{-1}(b) | v \geq r].$$

The conditional density above will, by definition of a cdf, depend on a constant  $c$  such that:

$$c \int_r^{\beta(\bar{v})} f_V(u) du = 1.$$

Since the antiderivative of  $f_V(u)$  is  $F_V(u)$  and  $F_V[\beta(\bar{v})] = 1$ , we have that

$$c = \frac{1}{[1 - F_V(r)]}.$$

Thus, the cdf reported in equation (6), is:

$$G_B(b) = \frac{1}{[1 - F_V(r)]} \int_r^v f_V(u) du = \frac{[F_V(v) - F_V(r)]}{[1 - F_V(r)]}.$$

Recalling that  $d\beta^{-1}(b)/db = 1/\beta'(v)$ , the pdf of observed bids of equation (7) is:

$$\begin{aligned} g_B(b) &= \frac{dG_B(b)}{db} = d \left\{ \frac{[F_V[\beta^{-1}(b)] - F_V(r)]}{[1 - F_V(r)]} \right\} / db \\ &= [1 - F_V(r)]^{-1} f_V[\beta^{-1}(b)] \frac{d\beta^{-1}(b)}{db} = \frac{f_V(v)}{[1 - F_V(r)] \beta'(v)}. \end{aligned}$$

### A.2 Stochastic dominance result

Let  $\hat{r}^*$  denote the optimal reserve price calculated using estimated functions  $\hat{F}_V(r)$  and  $\hat{f}_V(r)$ , and let  $r^*$  denote the true optimal reserve price calculated

using true functions  $F_V(r)$  and  $f_V(r)$ .

**Proposition** *If for all  $r$ ,  $\hat{F}_V(r) \geq F_V(r)$ , then  $\hat{r}^* \leq r^*$ .*

**Proof.** First, we note that the optimal reserve price,  $r^*$ , is the solution to

$$r - v_0 - \frac{[1 - F_V(r)]}{f_V(r)} = 0, \quad (20)$$

while we estimate  $\hat{r}^*$  by finding the value of  $r$  that satisfies

$$r - v_0 - \frac{[1 - \hat{F}_V(r)]}{\hat{f}_V(r)} = 0. \quad (21)$$

From (20) and (21), if  $\frac{[1 - \hat{F}_V(r)]}{\hat{f}_V(r)} \leq \frac{[1 - F_V(r)]}{f_V(r)}$ , then  $\hat{r}^* \leq r^*$ . Thus, we need to show that  $\frac{[1 - \hat{F}_V(r)]}{\hat{f}_V(r)} \leq \frac{[1 - F_V(r)]}{f_V(r)}$  implies  $\hat{F}_V(r) \geq F_V(r)$ . We proceed by noting that

$$\frac{[1 - \hat{F}_V(r)]}{\hat{f}_V(r)} \leq \frac{[1 - F_V(r)]}{f_V(r)} \Rightarrow -\frac{\hat{f}_V(r)}{[1 - \hat{F}_V(r)]} \leq -\frac{f_V(r)}{[1 - F_V(r)]}.$$

Since the exponential function is monotonically increasing, this implies that

$$\exp\left(\int_{\underline{v}}^r -\frac{\hat{f}_V(t)}{[1 - \hat{F}_V(t)]} dt\right) \leq \exp\left(\int_{\underline{v}}^r -\frac{f_V(t)}{[1 - F_V(t)]} dt\right). \quad (22)$$

Further, since

$$-\frac{f_V(r)}{[1 - F_V(r)]} = \frac{d}{dr} \log[1 - F_V(r)],$$

we have that the right-hand side of inequality (22) can be written:

$$\exp\left(\int_{\underline{v}}^r \frac{d}{dt} \log[1 - F_V(t)] dt\right) = [1 - F_V(r)]. \quad (23)$$

We have a similar expression for the left-hand side of (22). Consequently, from inequality (22) and equation (23), it follows that

$$[1 - \hat{F}_V(r)] \leq [1 - F_V(r)]$$

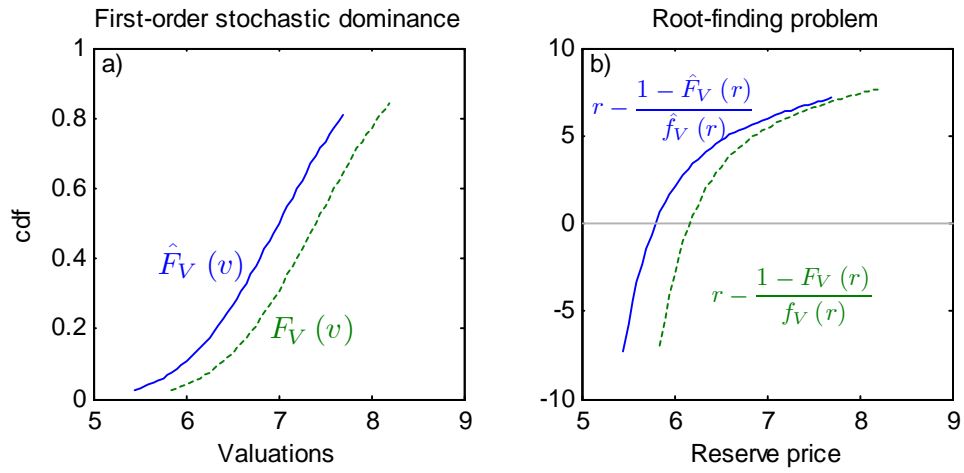
or

$$\hat{F}_V(r) \geq F_V(r).$$

■

Appendix B of Krishna [9] proved useful for the above proof. In figure 4, we illustrate the proof by sketching the relationship between the distribution functions in part a, and the corresponding root finding problem for an optimal reserve price in part b. As we see, and what was the point of the above proof, if  $\hat{F}_V(v) \geq F_V(v)$  for all  $v$ , then  $\hat{r}^* \leq r^*$ .

Figure 4: Illustration of stochastic dominance result



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